GEOMETRY - FINDING ANGLES

MATH CIRCLE (INTERMEDIATE) 02/24/2013

(1) **(Introduction)** Here are some basic facts that will help us calculate the angles of geometric figures...
   (a) The sum of the angles in any triangle is...
   (b) A pair of vertical angles are...
   (c) Angles lying along a straight line add up to...
   (d) An inscribed angle equals half the central angle which intercepts the same _____ of a circle.
   (e) Two inscribed angles intercepting the same arc of a circle are...
   (f) Rigid motions of the plane do not change...

(2) Angle bisector BK is drawn in isosceles triangle ABC, with $AB = AC$ and angle A equal to 36 degrees. Prove that $BK = BC$.

(3) Find the sum of the angles at the vertices of a five-pointed star.

(4) Can two angle bisectors in a triangle be perpendicular? Justify your answer.

(5) Chords AB and CD in circle S are parallel. Prove that $AC = BD$.

(6) The ratio of three consecutive angles in an inscribed quadrilateral is $2 : 3 : 4$. Find the measure of each angle.

(7) In triangle ABC, the measure of angle A is 90 degrees. Median AM, angle bisector AK, and altitude AH are drawn. Prove that angles MAK and KAH are congruent. (Note: A median connects a vertex with the midpoint of the opposite side of the triangle. An altitude connects a vertex with the opposite side of the triangle, intersecting it at a 90-degree angle.)

(8) Square $ABCD$ is given. A circle with radius AB and center A is drawn. This circle intersects the perpendicular bisector of BC in two points, of which O is the closest to C. Find the measure of angle AOC.

(9) Two circles intersect at points A and B. AC is a diameter of the first circle, and AD is a diameter of the second. Prove that points B, C, and D lie on the same straight line.

- Math Kangaroo Problems -

(10) In the picture, the points Q, S, and R are collinear, the measure of angle QPS is 12 degrees, and $|PQ| = |PS| = |RS|$. What is the measure of angle QPR?

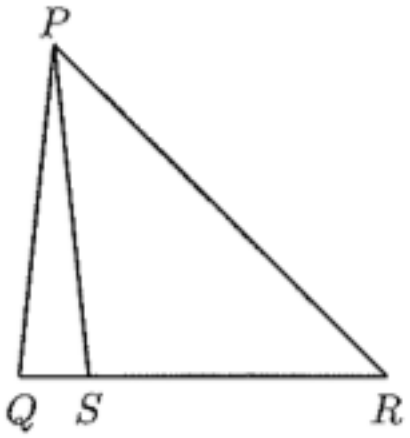

FIGURE 0.1. Problem 10.

(11) Quadrilateral $ABCD$ is a square (see the picture). The measure of $\angle OND$ is 60^{o}. What is the measure of $\angle COM$?

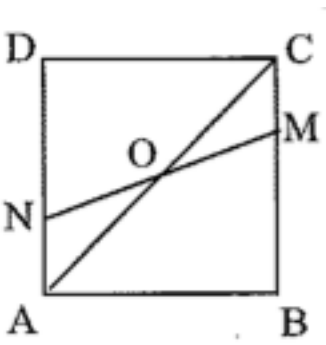

FIGURE 0.2. Problem 11.

(12) What fraction of the largest square is the shaded region, by area?

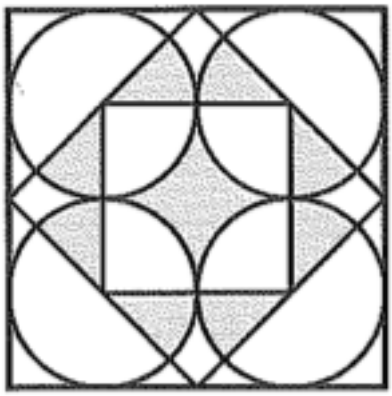

FIGURE 0.3. Problem 12.

(13) The figure in the picture is a regular nonagon (a regular 9-sided polygon). What is the measure of angle α?

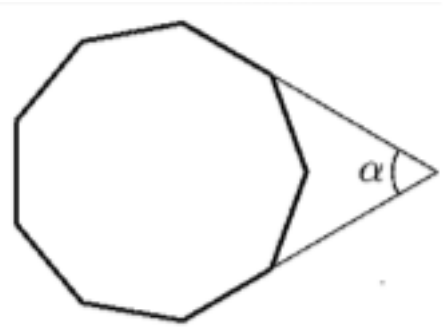

FIGURE 0.4. Problem 13.

(14) Triangles ABC and CDE are equilateral and congruent. If the measure of $\angle ACD = 80^{\text{O}}$,what is the measure of $\angle ABD$?

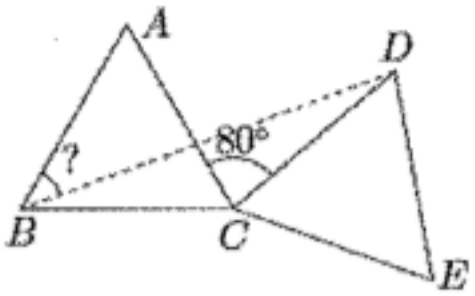

FIGURE 0.5. Problem 14.