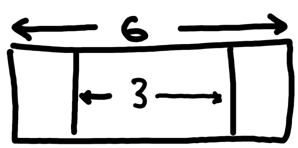
MATH KANGAROO

BEGINNER CIRCLE 2/3/2013

Problem 1. Basil wants to paint the slogan *VIVAT KANGAROO* on a wall. He wants to paint different letters different colors, and the same letters the same color. How many colors will he need?
Solution: This is just the number of different letters there are, which is 9

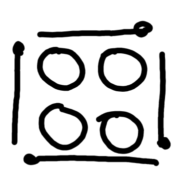
Problem 2. How many different numbers use the digits 1 through 5 exactly once. (For example: 13245, but not 1332445 or 1245)
Solution: There are $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ such numbers

Problem 3. A blackboard is 6m wide. The width of the middle part is 3m. The two other parts have equal width. How wide is the right hand part?



Solution: We know that the two blackboard sides total to a length of 3 meters. Because they are the same length, their individual length must be 1.5 meters.

Problem 4. Sally can put 4 coins in a square made using 4 matches. At least how many matches will she need in order to make a square containing 16 coins that do not overlap?



Solution: A square that has an area of 16 has a perimeter of 16, and each match is 2 units long, so 8 matches will do.

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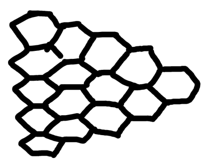
Problem 5. On a certain plane, the rows are numbered 1 to 25, but there is now row number 13. Row number 15 only has 4 passenger seats; all the other rows have 6 passenger seats. How many seats for passengers are there on this plane?
Solution: There are

$$23 \times 6 + 4 = 142$$

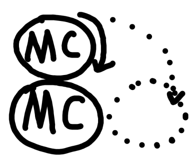
seats.

Problem 6. When it is 4 o'clock in the afternoon in London, it is 5 o'clock in the afternoon in Madrid and it is 8 o'clock in the morning on the same day in San Francisco. Ann went to bed in San Francisco at 9 o'clock yesterday evening. What was the time in Madrid at that moment?
Solution: 6PM

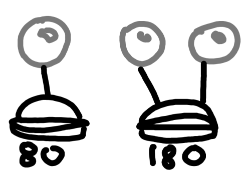
Problem 7. The picture on the right shows a pattern of hexagons. Draw a new pattern by connecting all of the midpoints of neighboring hexagons.



Problem 8. The upper coin is rotated around without slipping around the fixed lower coin to a position shown in the picture. Draw the resulting picture.



Problem 9. One balloon can lift a basket containing items weighing at most 80kg. Two such balloons can lift the same basket containing items weighing at most 180kg. What is the weight of the basket?



Solution: Let L be the lifting power of the balloon, and B the weight of the basket. Then

$$L = B + 80$$

$$2L = B + 180$$

tells us that

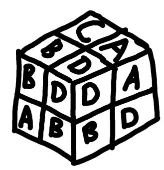
$$L = 100$$

and

$$B = 20$$

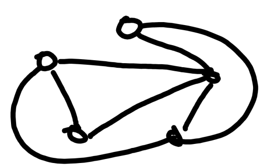
Problem 10. Vivien and Mike were given some apples and pears by their grandmother. They had 25 pieces of fruit in their basket altogether. On the way home Vivien ate 1 apple and 3 pears, and Mike ate 3 apples and 2 pears. At home they found out that they brought home the same number of pears as apples. How many pears were they given by their grandmother?
Solution: They end with 16 fruit, of with 8 are pears. Since they ate 5 pears, they must have started with 13 pears.

Problem 11. Lisa has 8 dice with the letters A, B, C, and D, with the same letter on all sides of each dice. She builds a block with them. Two adjacent dice always have different letters. What letter is on the die that cannot be seen in this picture?



Solution: As A, C, and D are the adjacent die, it must be B

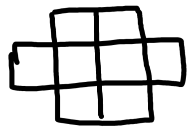
Problem 12. There are five cities in Wonderland. Each pair of cities is connected by one road, either visible or invisible. On the map of wonderland, there are only seven visible roads, as shown. Alice has magical glasses: when she looks at the map through these glasses she only sees the roads that are otherwise invisible. How many invisible roads can she see?



Solution: There are 10 roads total, and 7 are visible, so there must be 3 invisible roads.

Problem 13. The positive integers have been colored red, blue or green: 1 is red, 2 is blue, 3 is green, 4 is red, 5 is blue, 6 is green, and so on. Renate calculates the sum of a red number and a blue number. What colors can the resulting number be?
Solution: Using modular arithmetic, we say that red+blue$\equiv$ 1+2$\equiv 3$, which is green.

Problem 14. The perimeter of the figure to the right, made up of identical squares, is equal to 42cm. What is the area of the figure?

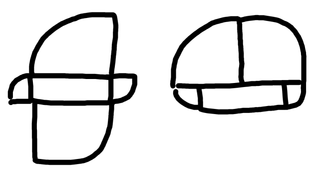


Solution: The side length must be 3 per square (as there are 14 line segments on the perimeter), and there are 8 squares. Each square has an area of 9, so

$$9 \times 8 = 72$$

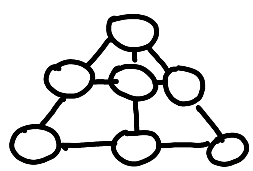
is the area.

Problem 15. Look at the pictures. Both shapes are formed from the same 5 pieces. The rectangle measures 5cm $\times$ 10 cm, and the other parts are quarters of two different circles. The difference between the lengths of the perimeters of the two chapes is what?



Solution: The small circles has a radius of 5, and the large circles a radius of 10. Both figures have the same number of curved components in their circumference, so the difference is going to come to the edges. The left diagram has 2 large radii and 2 small radii for an edge length of 30, while the right diagram has an edge length of only 10. Therefore, the difference in perimeter is 20cm.

Problem 16. Place the numbers from 1 to 7 in the circles in such a way that the sum of the number on each of the indicated lines of three circles is the same. What is the number at the top of the triangle?



Solution: The number placement is (from left to right, top to bottom) 4 1 6 5 7 2 3, so the number must be 4.

Problem 17. Isaac has a group of 7 students, and needs to select 4 of them for a class project. How many ways can he do this?
Solution: $\binom{7}{4} = \frac{7!}{4!2!} = 35$ ways

Problem 18. A rubber ball falls vertically through a height of 10m from the roof of a house. After each impact on the ground is bounces back up to $\frac{4}{5}$ of the previous height. How many times will the ball appear in front of a rectangular window whose bottom edge is at a height of 5m and whose top edge is at a height of 6m?
Solution: We need the ball to make a height less than 5, which it does after its fourth bounce, as

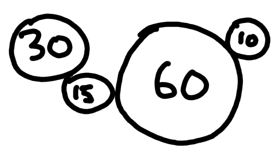
$$10 \times \left(\frac{4}{5}\right)^4 < 5 < 10 \times \left(\frac{4}{5}\right)^3$$

. It is seen passing the window twice on every bounce other than the last one, where it reverses direction in the middle of the window, as

$$5 \times \left(\frac{4}{5}\right)^3 < 6$$

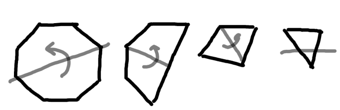
. Therefore, it is seen 6 times.

Problem 19. There are 4 gear wheels on fixed axles next to each other, as shown. The first one has 30 teeth, the second one 15, and the third one 60 and the fourth one 10. How many revolutions does the last gear wheel make when the first one goes through 1 revolution?



Solution: This in only dependent on the first and last gear, and so the ratio is $\frac{30}{10} = 3$ times

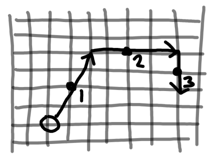
Problem 20. A regular octagon is folded in half exactly three times until a triangle is obtained, as shown. Then the apex is cut off at a right angle, as shown in the picture. Draw a picture of what the paper looks like when unfolded.

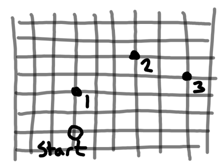


Problem 21. Jonathan is creating a math circle handout, which is to be 5 problems long. The three different types of problems he can write are Combinatorics problems, modular arithmetic problems, or graph theory problems. How many different kinds

of handouts can he write (it only matters how many problems of each kind he puts in his handout, not the order of the problems)
Solution: There are $\binom{5+3-1}{3}$ ways

Problem 22. Kangaroos Hip and Hop play jumping by hopping over a stone, then landing across so that the stone is the midpoint of the segment traveled during each jump. Picture 1 shows how Hop jumped three times hopping over stones marked 1,2, and 3. Hip has the configuration of stones marked 1,2, and 3 (to jump over in this order) but starts in a different place a shown on picture 2. Which point is his landing point?





Problem 23. There were twelve children at a birthday party. Each child was either 6, 7, 8, 9, or 10 years old, with at least one child of every age. Four of them were 6 years old. In the group the most common age was 8 years old. What was the average age of the twelve children?
Solution: Try writing out the party, starting with the guests you know (1 of each, and 4 sixes).

$$6, 6, 6, 6, 7, 8, 9, 10$$

the most popular age is 8, there must be at least 5 8's. However, that is already 12 guests! So the guests must be

$$6, 6, 6, 6, 7, 8, 8, 8, 8, 8, 9, 10$$

This makes the guest's on average 7.5.

Problem 24. Rectangle $ABCD$ is cut into four smaller rectangles, as shown in the figure. The four smaller rectangles have the properties:

(a) The perimeters of them are 11, 16 and 19
(b) The perimeter of the fourth is neither the biggest nor the smallest of the four.

What is the perimeter of the original rectangle $ABCD$?
Solution: We know that the perimeter of the big rectangle $\frac{1}{2}$ the sum of the perimeters of the four smaller rectangles. Let x_1 and x_2 be the sizes of the horizontal cuts and y_1 and y_2 be the sizes of the vertical cuts. Then $2(x_1 + y_1) = 16$, $2(x_1 + y2) = 12$, $2(x_2 + y_1) = 19$ and so the perimeter of the mystery rectangle

$$\begin{aligned} 2(x_2 + y_2) &= 2(x_2 + y_1) + 2(x_1 + y_2) - 2(x_1 + y_1) \\ &= 11 + 19 - 16 \\ &= 14 \end{aligned}$$

making the total perimeter 30

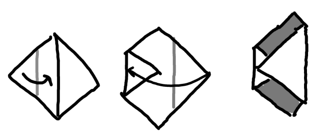
Problem 25. How many numbers have both of the following properties:

(a) They use the digits 1 through 5 once.
(b) No 3 consecutive digits appear in a row. (For instance, 41235 is not allowed)

Problem 26. Kanga wants to arrange the twelve numbers from 1 to 12 in a circle in such a way that any neighboring numbers always differ by either 1 or 2. Draw such an arrangement.
Solution: Start by laying out 12. The neighbors of 12 must be 11 and 10. The neighbor of 11 must be 9, and the neighbor of 10 must be 8, and so on and so forth, until you get a unique arrangement.

Problem 27. Peter wants to cut a rectangle of size 6×7 into squares with integer sides. What is the smallest number of squares he can get?
Solution: He can cut 5 squares, by making 1 4×4, 2 3×3 and 2 2×2 squares.

Problem 28. A square shaped piece of paper has an area of 64cm^2. The square is folded twice as shown in the picture. What is the sum of the areas of the shaded rectangles?



Solution: The long side of each shaded region has half the side length of the paper, so is 4 cm long. The short side of each shaded region is a quarter the side length of paper, and is therefore 2 cm long. This brings the total area of each rectangle to 8, for a total area of 16 cm squared.

Problem 29. Abid's house number has 3 digits. Removing the first digit of Abid's house number, you obtain the house number of Ben. Removing the first digit of Ben's house number, you get the house number of Chiara. adding the house numbers of Abid, Ben and Chiara gives 912. What is the second digit of Abid's house number?
Solution: Let A be the first digit of Abid's, B be the first digit of Ben's house, and C the first digit of Chiara's house. Then

$$100A + 20 + B + 3C = 912$$

This says that $C = 4$. Then $B = 5$, and finally $A = 8$. So the second digit must be 5.

Problem 30. I give Ann and Bill two consecutive positive integers (for instance 7 to Ann and 6 to Bill). They know their numbers are consecutive, they know their own number, but they do not know the number that I gave to the other one. Then I heard the following discussion. Ann said to Bill: "I don't know your number." Bill said to

Ann: "I don't know your number." Then Ann said to Bill: "Now I know your number! It is a divisor of 20!" What is Ann's number?

Solution: Since Ann's number is both one more and one less than a divisor of 20, and the only divisors of 20 that are 2 apart are 2 and 4, Ann's number must be 3.