ORMC Intermediate **July 120th, 2025**

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Warm-up

Problem 1 *What you see below is a part of the decimal multiplication table, in random order and in Japanese. Find the missing factors denoted by question marks.*

futatsu $\times$ *yottsu* $=$ *yattsu*

itsutsu $\times$ *itsutsu* $=$ *ni juu go*

yattsu $\times$ *kokonotsu* $=$ *nana juu ni*

itsutsu $\times$ *yattsu* $=$ *yon juu*

mittsu $\times$ *mittsu* $=$ *kokonotsu*

kokonotsu $\times$ *mittsu* $=$ *ni juu nana*

muttsu $\times$ *mittsu* $=$ *juu hachi*

futatsu $\times$ *rei* $=$ *rei*

kokonotsu $\times$ *?* $=$ *hachi juu ichi*

muttsu $\times$ *kokonotsu* $=$ *?*

yottsu $\times$ *?* $=$ *san juu ni*

futatsu $\times$ *nanatsu* $=$ *?*

Algebra of statements

A *statement* is an expression which is either *True* or *False*. For example, "Let's go!" is not a statement, while "My math teacher is not human!" is.

Problem 2 *In the space below, write two sentences that are statements in the above sense and two more that are not.*

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-
-
-

If a statement A is true, we write $A = 1$. If a statement A is false, we write $A = 0$.

Problem 3 *Determine which of the sentences below are statements and find their values.*

A 23 *is divisible by* 5.

B *Please don't smoke on board the aircraft.*

C $7x + 5y = 70$

D *Pyotr Tchaikovsky is a famous Russian hockey player.*

E *What time is it now?*

F *Get out of here!*

G *Math is fundamental for understanding all other sciences.*

If a statement mentions only one event, true or false, it is called *simple*. If a statement mentions more than one event, it is called *composite*. For example, the statement *I come to the Math Circle by car* is simple, while the statement *I come to the Math Circle by car or by bus* is composite.

Let A and B be statements. Let us define $A+B$ as the statement A *or* B. For example, if $A =$ *three is greater than two* and $B =$ *three is greater than five*, then $A + B =$ *three is greater than two or than five.*

The statement A *or* B is false if and only if both A and B are false. If either of the statements A or B is true, then A *or* B is true as well.

A	B	$A+B$
0	0	0
1	0	1
0	1	1
1	1	1

We can see from the above *truth table* that in the algebra of logic $0 + 0 = 0$, while $1 + 0 = 0 + 1 = 1 + 1 = 1$.

Problem 4 *Is the logical addition commutative? Why or why not? Please write down your explanation in the space below.*

Problem 5 *Prove that* $A + 0 = A$ *and* $A + 1 = 1$.

Give a verbal interpretation to the above algebraic statements.

Problem 6 *Prove that* $\underbrace{A + A + \ldots + A}_{n \text{ times}} = A$.

Problem 7 *Form the logical sum of the following three statements and find its value.*

$A =$ *The planet of Earth rotates around the North Star.*

$B =$ *The planet of Earth rotates around Alpha Centauri.*

$C =$ *The planet of Earth rotates around the Sun.*

$A + B + C =$

Problem 8 *Prove that for the logical addition,* $(A + B) + C = A + (B + C)$. *Hint: use the truth table below.*

A	B	C	$A+B$	$B+C$	$(A+B)+C$	$A+(B+C)$
0	0	0				
1	0	0				
0	1	0				
0	0	1				
1	1	0				
1	0	1				
0	1	1				
1	1	1				

Similar to the logical addition, we can introduce logical multiplication. Let us define $A \times B$ as the statement A *and* B. For example, if $A =$ *three is greater than two* and $B =$ *three is greater than five*, then $A \times B =$ *three is greater than two and five*. Quite obviously, A *and* B is true if and only if both A and B are true.

A	B	$A \times B$
0	0	0
1	0	0
0	1	0
1	1	1

In the algebra of logic just like in the algebra of numbers, $0 \times 0 = 1 \times 0 = 0 \times 1 = 0$, while $1 \times 1 = 1$.

Problem 9 *Is the logical multiplication commutative? Why or why not? Please write down your explanation in the space below.*

Problem 10 *Prove that* $A \times 0 = 0$ *and* $A \times 1 = A$.

Problem 11 *Prove that* $\underbrace{A \times A \times \ldots \times A}_{n \text{ times}} = A$

Problem 12 *Prove that for the logical multiplication,* $(A \times B) \times C = A \times (B \times C)$.

A	B	C	$A \times B$	$B \times C$	$(A \times B) \times C$	$A \times (B \times C)$
0	0	0				
1	0	0				
0	1	0				
0	0	1				
1	1	0				
1	0	1				
0	1	1				
1	1	1				

Problem 13 *Form the logical product of the following three statements and find its value.*

$A =$ *Lobsters live in the ocean.*

$B =$ *Mobsters do not live in the ocean.*

$C =$ *Lobsters are no better than mobsters.*

$A \times B \times C =$

Problem 14

$A =$ *I start to like the science of logic.*

$B =$ *"To Kill a Mockingbird" is a hunters' guidebook.*

$C = 48$ *is divisible by* 12.

Form the following statements from the above A, B, *and* C *and find their value.*

$A \times (B + C) =$

$A \times B + A \times C =$

Problem 15 *Prove that*
$A \times (B + C) = (B + C) \times A = A \times B + A \times C$.

A statement A preceded by *it is not true that ...* or a statement equivalent to such is called the *negation* of A and is denoted as $\neg A$. For example, the negation of the statement B from Problem 14 reads as follows. *"To Kill a Mockingbird" is not a hunters' guidebook.* The following is the truth table for the negation.

A	$\neg A$
0	1
1	0

Problem 16 *Write down the negation of the statement "I come to the Math Circle by car or by bus" in the space below.*

Problem 17 *Write down your own composite statement and its negation.*

Statement:

Negation:

Problem 18 *Can the statement $A + \neg A$ be false? Why or why not?*

Write the fact down as an algebraic formula.

Problem 19 *Can the statement $A \times \neg A$ be true? Why or why not?*

Write the fact down as an algebraic formula.

Problem 20 *Find the following.*

$\neg\neg A =$

Problem 21 *Given the statements*

A: Bob is driving to work.

B: Bob is shaving.

C: Bob is eating a burger.

form the following statements.

- $AB + \neg C =$

- $(A + B)C =$

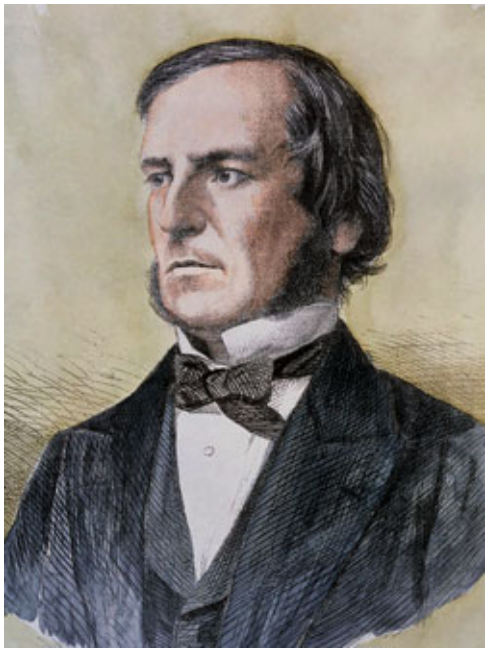
- $A\neg B + C =$

- $\neg A\neg BC =$

Problem 22 *Using the simple statements A, B, and C from Problem 21, rewrite the following as a mathematical formula.*

It is not true that Bob is either driving to work and shaving or eating a burger.

The algebra of logic we have studied above is called *Boolean*, after George Boole (1815-1864), an English mathematician, philosopher and logician.



George Boole

Below you will find one more feature of the algebra that truly distinguishes it from every other algebraic structure you have seen before.

In Problem 15, we have proven that, similar to the algebra of numbers, multiplication in Boolean algebra is distributive.

$$A \times (B + C) = A \times B + A \times C$$

In Boolean algebra, unlike the algebra of numbers, addition is distributive with respect to multiplication as well!

$$A + (B \times C) = (A + B) \times (A + C) \tag{1}$$

Problem 23 *Prove formula 1.*

Problem 24 *Is the sentence "this statement is false" a statement? If you think it is, find its value. If you think it's not, give a reason.*

If you are finished doing all the above, but there still remains some time ...

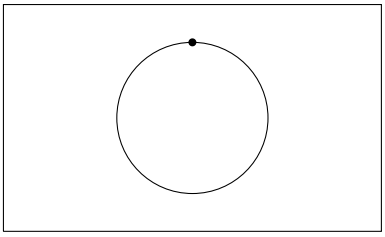
... let us study the *wagon wheel effect* ...

Problem 25 *A film runs through the projector at the rate of sixteen frames per second. On the screen, you see a moving car. In real life, the diameter of the car's tires is one meter. In the movie, the car's wheels rotate four times per second. What is the real life speed of the car?*

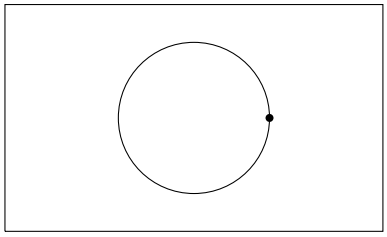
Here is the beginning of the solution. In the movie, the car's wheels rotate four times per second. The time period between the frames is

$$\tau = \frac{1}{16} \; sec.$$

If during this time period of time in real life the wheels rotate n full times and a quarter, then we see the wheels rotating the right way on the screen.

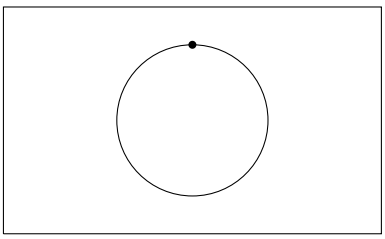


Frame 1

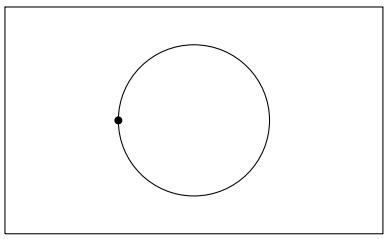


Frame 2

If the number of the rotations during the time period τ is n full times and three quarters, then we see the wheels rotating the wrong way.



Frame 1



Frame 2

Please finish the solution.