

Keakeya's Squeegee (Warm-Up)

Prepared by Matthew Kowalski on April 17, 2025
 Inspiration taken from Mathologer's "The Keakeya needle problem (the squeegee approach)"

Instructor's Handout

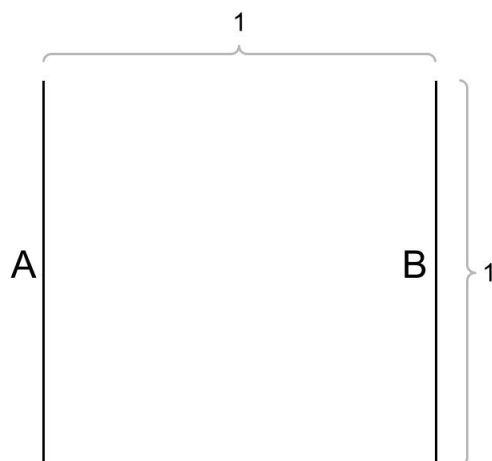
Suppose that you are cleaning a window. You have your squeegee (a line segment of length 1) placed on line A and you want to move it to line B (distance 1 from A).

However, in an act of rebellion, you want to *clean the smallest area of window possible*.

What is the smallest area you can come up with? How do you do it?

Note: we consider the window to be infinite, so you can move the squeegee anywhere.

Hint: You can rotate the squeegee.



Solution

The area you clean can be made arbitrarily small.

Rotate the squeegee a tiny angle $\theta \ll 1$ clockwise around its bottom point. This will clean out a region of area $\theta/2$.

Slide the squeegee upwards in the direction it is pointing, until the top of the squeegee is lined up with B . Because the squeegee has no width, this does not clean any area.

Rotate the squeegee θ counter-clockwise around its top point. This will again clean a region of area $\theta/2$.

Slide the squeegee downwards until it is lined up with B . The total area you cleaned is θ and θ can be made arbitrarily small. (*A downside to this method is that you need a very large window. What if you're confined to a window of fixed area?*)