

# Möbius Strips, Dwarves, and Fake Coins

Early Elementary Meeting 2: 1/20/13

## Part I

### Möbius Strips Continued

1. Mark a point on a cylinder such that it has the same distance to both edges. After that, draw a line on the cylinder so that all the points on the line are the same distance to both edges.

(a) What shape is this line?

- (b) Can you draw another line on the cylinder such that all the points on this line are at the same distance from both edges? (HINT: Think about how many sides a cylinder has.)

What is the shape of this line?

- (c) Do the two lines above intersect?

2. Mark a point on a Mobius strip such that it has the same distance to both edges. After that, draw a line on the Mobius strip so that all the points on the line are the same distance to both edges.

Are there any other points on the Mobius strip that are the same distance to the edges?

3. (Carefully!) cut a cylinder along the middle line. (You can use the cylinder you have already drawn the middle line on). What do you get as a result?

4. (Carefully!) cut a Mobius strip along the middle line. What do you get as a result?

5. Take another Mobius strip and draw a line which is twice as close to one of the edges compared to the other.

(a) What do you think will happen if you cut the Mobius strip along this line? (NOTE: Make a prediction, don't cut yet!)

(b) Cut the Mobius strip along this line and see what you get.

## Part II

# Snow White and the 7 dwarves

1. Snow White and the 7 dwarves are playing a game:

- Snow White wrote the number 20 on a piece of paper and gave it to the dwarves;
- Each of the 7 dwarves looks at the number on the paper and adds or subtracts 1 to it;

Then he gives the number he gets to the next dwarf.

(a) Can they get 21 in the end? (If yes, show how. If not, explain why not.)

(b) Can they get 23?

(c) Can they get 19?

(d) Can they get 22?

(e) Can they get 17?

(f) Can they get 14?

(g) Can you list all the numbers they can get?

2. Find the missing number:

66	99
61	19
86	

Explain the pattern:

## Part III

# Find the fake coins in one weighing



In the problems below, each real coin weights 10 grams. Each fake coin weights 9 grams.

1. Suppose you have three 2 bags full of coins so that
  - one of the bags has fake coins;
  - the other bag has real coins;

How can you determine which bag has real coins and which has fake coins *in just one weighing*?

- (a) Suppose you put the coin(s) from only one of the bags on the scale. Will you be able to determine which bag contains the fake coins?
- (b) Suppose you put one coin from each of the bags on the scale. Will you be able to determine which bag contains fake coins?
- (c) Decide how many coins from each of the bags you need to put on the scale; describe the possible outcomes (how much could the coins weight together), and explain how you can determine which bag has the fake coins.

2. Now suppose you have 3 bags full of coins. Again, one of the bags holds fake coins. How can you determine which bag has fake coins in just one weighing? Describe what you will put on the scale and how you will decide where the fake coins are.

3. Suppose now you have 10 bags of coins. One of the bags is filled with fake coins. Can you figure out which bag has the fake coins in just one weighing?

4. Ten thousand pine trees grow in a forest. Scientists have discovered that in this forest, no pine tree has more than 1000 pine needles on it. Show that one can find two pine trees in the forest so that they have the same number of pine needles.

5. Bacteria are growing in a laboratory dish. After each hour, every bacteria divides into two, to make two new ones. After 10 hours, the dish is full.

When was it half full?

6. When was it one quarter full?