LAMC Junior Circle **January 27, 2013**

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Word problems on fractions

Problem 1 *Nadya thinks of a number such that the sum of one third of the number and one fourth of the number equals twenty one. What number does Nadya think of?*

Problem 2 *Victor had five friends at his birthday party. He gave his first friend one sixth of the birthday cake. The second kid got one fifth of what was left. The third friend got one fourth of the remains, and the fourth got one third of what was left after that. Finally, Victor took a half of what was left of the cake for himself and gave the other half to his fifth friend. Is this a fair way to divide a cake between the friends? If not, who got the biggest part of the cake?*

Problem 3 *A fraction p/q has the following properties:*
1. it is positive and proper, $0 < p/q < 1$;
2. the numerator, p, and the denominator, q, have no common factors; and
3. the fraction value does not change, if one adds 30 to the numerator and 40 to the denominator. What is the fraction?

Problem 4 *Let p_1/q_1 and p_2/q_2 be positive proper fractions such that*
1. p_1 and q_1 have no common factors and neither have p_2 and q_2;
2. $q_1 \neq q_2$, $q_1, q_2 > 100$.
Is it possible that the sum of the fractions

$$\frac{p_1}{q_1} + \frac{p_2}{q_2}$$

is a fraction such that its denominator is less than 100?

Probability

Definition 1 *The set of all the possible outcomes of an experiment is called the sample space.*

Let us represent the result of rolling two dice as a two-digit number with the first digit coming from the first die and the second digit coming from the second. For example, if we roll 3 and 5, we write down the result as 35, and if we roll 5 and 3, we write down the result as 53.

Problem 5 *Represents the sample space for rolling two dice as a* 6×6 *table.*

Problem 6 *You roll two dice. What is the chance of getting a double?*

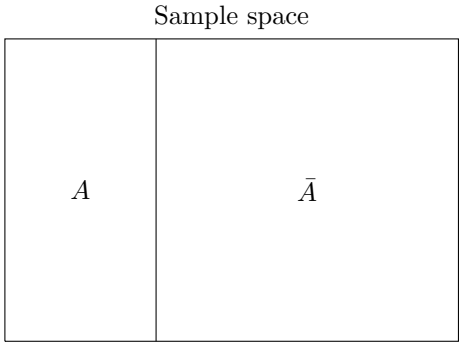
Problem 7 *You roll two dice. What is the chance that the sum is greater than 10?*

Problem 8 *You roll two dice. What is the chance that the sum does not exceeds 10? Can you use the previous problem to figure out the probability for this one? Why or why not?*

Definition 2 *For an event A, the event* **A does not occur** *is called complementary to A and is denoted as $\bar{A}$.*

For example, the events from Problems 7 and 8 are complementary to each other. The probabilities of complementary events add up to one.

$$P(A) + P(\bar{A}) = 1 \tag{1}$$



Problem 9 *You roll two dice. $A = \{$The total score you get is less than 9.$\}$ List all the elementary events constituting $\bar{A}$. Find $P(\bar{A})$. Use $P(\bar{A})$ to find $P(A)$.*

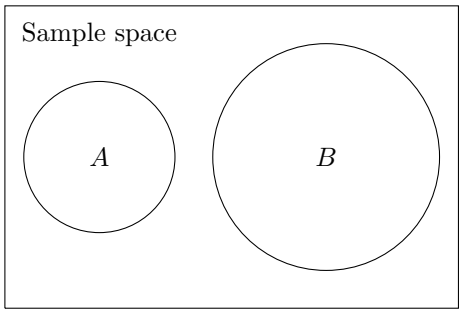
Problem 10 *You toss a coin 10 times. What is the size of the sample space?*

Problem 11 *You toss a coin 10 times. How likely are you to get at least one tail?*

Problem 12 *You choose a number from the set* $1, 2, 3, \ldots, 100$ *at random. What is the chance that you don't get a perfect square?*

Problem 13 *You pull a single card out of a deck of 52 cards. What is the chance that the card is not a King?*

Definition 3 *The events A and B are mutually exclusive if $A \cap B = \emptyset$ (the intersection of the sets A and B is empty).*



In other words, an elementary event can either be in the set A or in the set B, but not in both. For example, complementary events are mutually exclusive. Here is one more example. You toss a coin twice. The events $A = TT$ and $B = HH$ are mutually exclusive, but not complementary.

Problem 14 *Give your own example of two mutually exclusive events that are not complementary.*

Let two events A and B be mutually exclusive. Then

$$P(A \textit{ and } B) = 0,$$
$$P(A \textit{ or } B) = P(A) + P(B). \tag{2}$$

Problem 15 *You toss a coin 4 times. What is the likelihood that you get either all heads or all tails?*

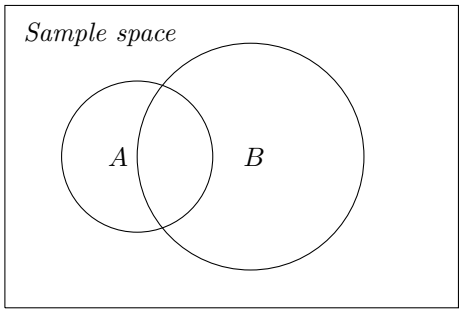
Problem 16 *You toss a coin 4 times. What is the likelihood that you get either all heads or at least 3 tails?*

Problem 17 *Give an example of two probabilistic events A and B such that the event $\{A \text{ or } B\} = A$.*

Problem 18 *Prove the following formula for two generic (not necessarily mutually exclusive) events A and B.*

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \tag{3}$$

Hint: the below picture should help.



Problem 19 *You toss a coin 4 times. What is the chance that you get either at least 1 head or at least 1 tail?*

Definition 4 *In probability theory, two events are called independent, if the occurrence of one does not affect the probability of the other.*

For example, two tosses of a coin are independent events. No matter what the result of one toss is, it does not affect the other. For two independent events,

$$P(A \text{ and } B) = P(A)P(B). \tag{4}$$

Problem 20 *You toss a coin 1,000 times and get heads all the 1,000 times. What is the chance that you get the heads again for the 1,001st toss?*

Problem 21 *You roll two dice twice. What is the chance that you get doubles both times? Hint: take a look at Problem 6.*

Problem 22 *You pull two cards out of the deck of 52. What is the chance that you get the same suit?*

Problem 23 *You have 10 cards, each with a digit, zero through ten. You take two cards, a and b, at random one after another, and form the two-digit number* $10a + b$*. (In this problem, we consider the resulting number as a two-digit even in the case* $a = 0$*.) What is the chance that the resulting two-digit number is divisible by 18?*