

Dr. Oleg Gleizer
prof1140g@math.ucla.edu

Arithmetic sequences and random walks

Let a and $b \neq 0$ be numbers. The sequence $x_0 = a$, $x_1 = a + b$, $x_2 = a + 2b$, ... $x_n = a + nb$, $n \in \mathbb{N}$, is called an *arithmetic sequence*.

Problem 1 Show that the sequence of numbers $1, 2, 3, \dots, n$; $n \in \mathbb{N}$, is an arithmetic sequence. What are the a and b ?

Answer: $a =$ _____ , $b =$ _____

Problem 2 Find the sum $S = 1 + 2 + 3 + \dots + n$, $n \in \mathbb{N}$.

Answer: $S =$ _____

Problem 3 Let x_0, x_1, \dots, x_n be an arithmetic sequence. Find the sum

$$\sum_{i=0}^n x_i = x_0 + x_1 + x_2 + \dots + x_n$$

in a closed form.

Answer: $S =$ _____

Problem 4 Alice invests \$1,000 in a bond that matures in three years and pays 5% annually. How much money will Alice have in the end, if she does not spend or re-invest the interest?

Answer: Alice will have \$ _____ in the end.

Problem 5 Let the numbers x_{n-1} , x_n , and x_{n+1} be three consecutive members of an arithmetic sequence. Show that

$$x_n = \frac{x_{n-1} + x_{n+1}}{2}.$$

Problem 6 Each of the numbers of the sequence x_0, x_1, \dots, x_n , except for x_0 and x_n , is the average of the two neighboring numbers. Show that the sequence is arithmetic. What are the a and b ?

Answer: $a =$ _____ , $b =$ _____

Problem 7 Bob has \$5. Passing by a Ferrari dealership in Beverly Hills, Bob sees a Ferrari car with the selling price of \$260,000. Bob wants to buy the car.

Charlotte runs an on-line casino. Each bet is \$1 with a 50% chance to win or lose. Bob starts gambling hoping to get enough money to purchase the Ferrari.

At the end of the problem, you will be able to answer the following two questions.

- (1) What is Bob's chance to achieve his financial goal?
- (2) What is the expected number of gambles Bob would need to buy the car?

The problem continues to the next page.

Let n be an integer, $0 \leq n \leq 260,000$. Let p_n be the probability of Bob achieving his goal, given that he has n dollars.

- What is p_0 ?

$$p_0 = \underline{\hspace{2cm}}$$

- What is $p_{260,000}$?

$$p_{260,000} = \underline{\hspace{2cm}}$$

- What is p_n , given that you know p_{n-1} and p_{n+1} ?

$$p_n = \underline{\hspace{4cm}}$$

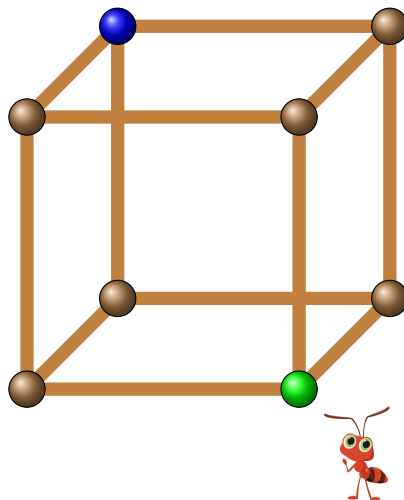
- What is the chance that Bob achieves his financial goal?

$$p_5 = \underline{\hspace{2cm}}$$

- What is the expected number of gambles, g_5 , Bob needs to take to buy the car? Hint: each time Bob bets, the number of the gambles is increased by one.

$$g_5 = \underline{\hspace{2cm}}$$

Problem 8 *An ant moves randomly on a cube made of one-unit-long craft sticks and play-doh. The insect starts from the green vertex of the cube, please see the picture below. The ant will stop when/if it reaches the opposite vertex, colored blue on the picture.*



- *What is the chance that the ant never stops?*
- *What is the expected length of the ant's travel?*

Hint: consider all the vertices of the cube having the same distance from the terminal vertex as one state of the game.