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### Fractions

#### Problem 1

$$\frac{2}{3} - \frac{1}{2} =$$

$$\frac{2}{3} - \frac{1}{3} =$$

$$\frac{2}{3} - \frac{1}{4} =$$

$$\frac{2}{3} - \frac{1}{5} =$$

$$\frac{2}{3} \times 3 \div 5 =$$

$$\frac{2}{3} \div 5 \times 3 =$$

$$\frac{2}{3} \times \frac{3}{5} =$$

$$\frac{2}{3} \div \frac{5}{3} =$$

**Problem 2** Put the correct sign,  $>$ ,  $<$ , or  $=$ , in the box between the numbers. No guessing please. Show your work.

$$\frac{15}{16} \square \frac{16}{17}$$

$$\frac{7}{8} \square \frac{15}{17}$$

$$\frac{8}{3} \square \frac{5}{2}$$

## Probability

A *probability*, also known as a *chance*, is a number showing how likely some event is to happen in an experiment. Let us call the event  $x$ . Then the probability of  $x$  taking place,

$$P(x) = \frac{\text{The number of the outcomes such that } x \text{ happens.}}{\text{The number of all the possible outcomes.}}$$

Note that by its very definition,  $0 \leq P(x) \leq 1$ .

In the following problems, we will be flipping a coin. Let us denote as  $H$  the event of the coin landing heads up and let us denote as  $T$  the event of the coin landing tails up.

**Problem 3** *You flip a coin once. What is the probability of getting the tail?*

$$P(T) =$$

**Problem 4** *You flip a coin twice. List all the possible outcomes.*

**Problem 5** *You flip a coin twice. What is the probability of getting the tails both times?*

$$P(TT) =$$

**Problem 6** *You flip a coin twice. List all the possible outcomes such that the tails occur at least once.*

$$x = \{ \quad \quad \quad \}$$

*What is the chance that you get the tails at least once?*

$$P(x) =$$

*What is the chance that you never get the tails in two flips?*

*What number do the last two probabilities add up to? Why?*

**Problem 7** *You flip a coin 1,000 times. How many times do you expect to get the heads?*

**Problem 8** *You flip a coin 1,000 times. Is 502 heads a likely output? How about 52 heads? Why?*

**Problem 9** *You roll a single die. List all the possible outcomes.*

**Problem 10** *You roll a single die. What is the chance of rolling an even number?*

**Problem 11** *You roll a single die. What is the chance that the number you get does not exceed two?*

**Problem 12** *You roll a single die. What is the chance that the number you get does not exceed five?*

**Problem 13** *You roll a single die. What is the chance that you roll a six? What number do the probabilities from Problems 12 and 13 add up to? Why?*

**Problem 14** *A wooden cube is painted and cut into 1,000 smaller cubes of equal size.*

- *How many of the little cubes will have three sides painted?*
  
- *How many of the little cubes will have two sides painted?*
  
- *How many of the little cubes will have one side painted?*
  
- *The little cubes are carefully mixed in a non-transparent bag. Once the mixing is finished, you pull out one of them. What is the chance that the cube you pull out has no paint on it?*

**Problem 15**

- *How many two-digit numbers are there? (A two-digit number cannot have zero as the first digit.)*
  
- *How many squares of two-digit numbers have one as their last digit?*
  
- *What is the probability that a square of a randomly taken two-digit number ends up with one?*

**Problem 16** *Create and solve your own probability problem.*