

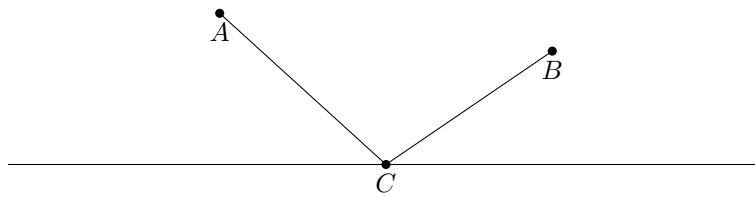
Geometric Optimization

Prepared by Mark on October 31, 2024
Based on a handout by Nakul & Andreas

Part 1: Optimization

Problem 1:

Let A and B be two points on the same side of a given line ℓ .
Find a point C on ℓ so that $|AC| + |BC|$ is minimized.



Definition 1:

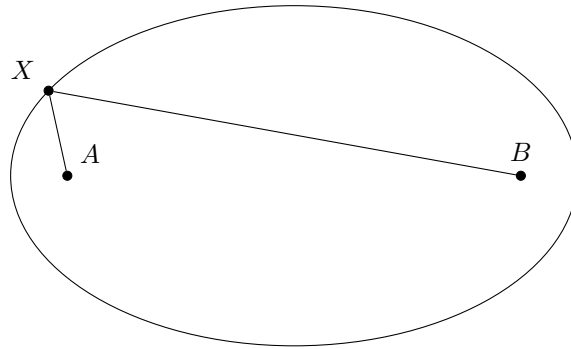
An *ellipse* with foci A , B and radius r is the set of all points C where $|AB| + |BC| = r$.

Problem 2:

Consider a reflective ellipse with foci A and B .

Find all points X on the ellipse where A can aim a laser at so that the beam reaches B .

Hint: use Problem 1



Problem 3:

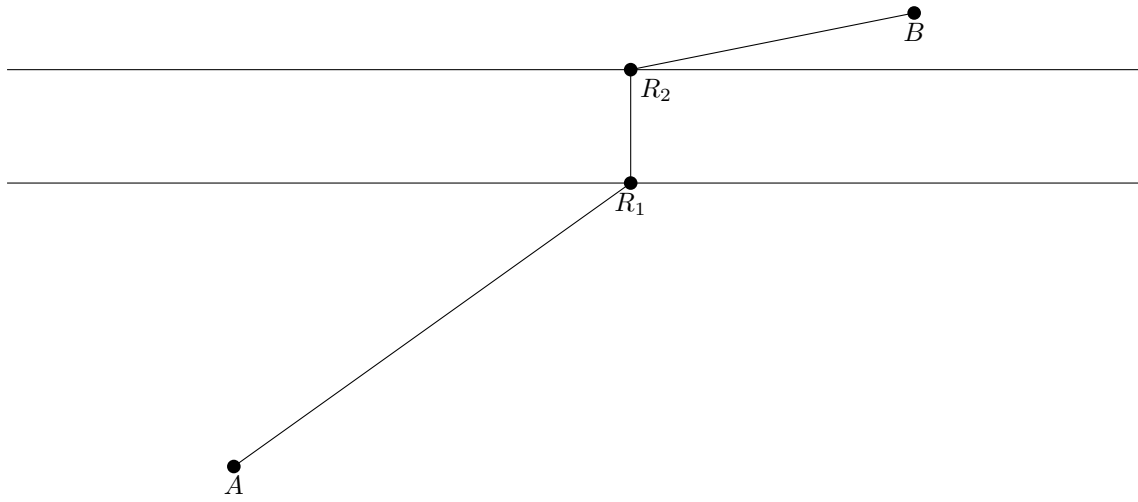
Let C be a point in the interior of a given angle. Find points A and B on the sides of the angle such that the perimeter of the triangle ABC is a minimum.

Problem 4:

In a convex quadrilateral $ABCD$, find the point T for which the sum of the distances to the vertices is minimal.

Problem 5:

A road needs to be constructed from town A to town B, crossing a river, over which a perpendicular bridge is to be constructed. Where should the bridge be placed to minimize $|AR_1| + |R_1R_2| + |R_2B|$?

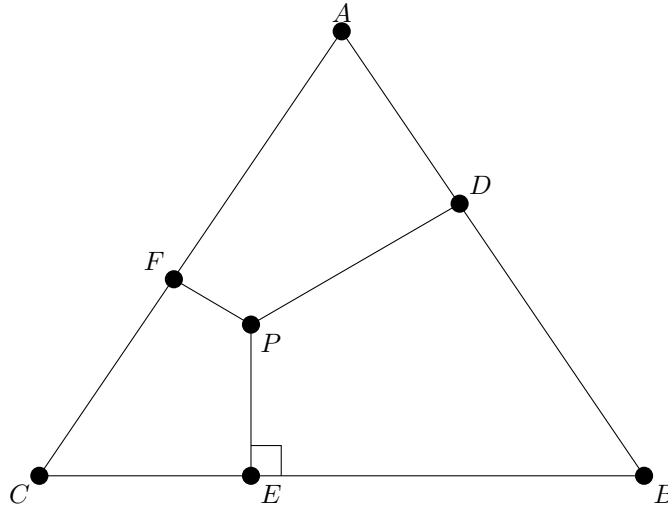


Problem 6:

Consider an equilateral triangle with vertices labeled A , B , and C .

Let P be a point inside this triangle. Place D , E , and F so that PD , PE , and PF are the perpendiculars from P to the sides of the triangle.

Find all points P where $|PD| + |PE| + |PF|$ is minimized.



Problem 7:

With the same setup as Problem 6, find all points P where $|PA| + |PB| + |PC|$ is minimized.

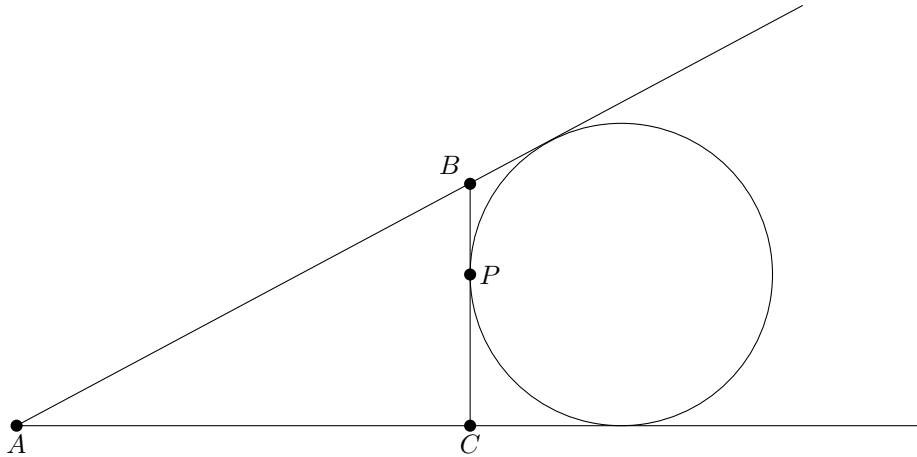
Problem 8:

Solve Problem 7 for a triangle that isn't equilateral.

Problem 9:

Draw a circle, then draw two distinct tangents ℓ_1 and ℓ_2 that intersect at point A .

Let P be a point on the circle between the tangents, and BC be the tangent at that point. Describe how P should be selected in order to minimize the perimeter of triangle ABC .



Problem 10:

Now, assume that ℓ_1 and ℓ_2 intersect at A , and pick a point P between them.

Find BC through P so that the perimeter of ABC is minimized.

Part 2: Bonus Problems

Problem 11:

Given a cube $A_1B_1C_1D_1A_2B_2C_2D_2$ with side length l , find the angle and distance between lines A_1B_2 and A_2C_1 .

Problem 12:

Consider a cube $A_1B_1C_1D_1A_2B_2C_2D_2$, and let K , L , and M be midpoints of the edges A_2D_2 , A_1B_1 , and C_1C_2 . Show that the triangle formed by KLM is equilateral, and that its center is the center of the cube.

Problem 13:

Consider all n -gons with a certain perimeter. Show that the n -gon with maximal area has equal sides

Problem 14:

Consider all n -gons with a certain perimeter. Show that the n -gon with maximal area has equal angles