

# LAMC Monthly Contest

Advanced Group

December 9, 2012

1. Find all three digits  $a, b, c$  from  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$  with the property

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = a.bcccccc\dots$$

2. Let  $P$  a point be inside the square,  $ABCD$  such that  $PA = 1$ ,  $PB = \sqrt{2}$ ,  $PC = \sqrt{3}$ .

- a). Find the length of the line segment  $PD$ .  
b). Find the angle  $\angle APB$ .

3. Two friends, Tom and Bill, play together the following game: Tom chooses a number from  $\{0, 1, \dots, 999\}$ , which he multiplies it by a factor of 2, then Bill adds 22 to the new result. Then Tom multiplies the new result by 2, and then Bill adds 22, and so on. One loses if he is the first one to obtain a number greater or equal to 1000. How many possibilities does Tom have to choose the first number in order to win the game?

4. Consider two positive integers  $m, n$  such that the numbers

$$\frac{m^2 + 2n}{n^2 - 2m} \text{ and } \frac{n^2 + 2m}{m^2 - 2n}$$

are integers.

- a). Show that  $|m - n| \leq 2$ .  
b). Find all pairs  $(m, n)$  with the above property.

5. Let  $p, q$  be two real positive numbers such that

$$\frac{1}{p} + \frac{1}{q} = 1.$$

Show that

$$\frac{1}{3} \leq \frac{1}{p(p+1)} + \frac{1}{q(q+1)} < \frac{1}{2} \text{ and } \frac{1}{p(p-1)} + \frac{1}{q(q-1)} \geq 1.$$

**All the problems are due January 13th, 2013. Please follow all the rules below. Good luck!**

If you plan to submit answers to the monthly contest, please follow the rules below.

1). This contest is proof based, i.e. you have to write the whole proof in order to get the whole 10 points for the problem, not only the answer ( Only writing down the answer will receive 1-2 points).

2). Please present all the reasoning and the proofs in logical order.

3). Each problem should be on a different page. If the question asks for a short answer, please highlight the answer.

4). Please submit either typed or neatly handwritten solutions.

5). You will receive partial credit for an unfinished proof. So you should write down any worthwhile ideas you have. However, your work should be legible, and you should not turn in any scratch paper.

6). You should explicitly state each theorem that you are using and also how you are using them. When quoting a theorem of a mathematician that has many theorems in math attributed to him/her, you should also write down the statement of the theorem (e.g. Cauchy's theorem).

7). You should obey all the rules above when writing the solutions, or else you will receive 0 points.

For questions, you can contact me at **franciscdodo@yahoo.com**.