

# Fun with Water Crossings

Math Circle (Intermediate)

December 2, 2012

**SPECIAL DIRECTIONS:** These are not trick questions. Assume there is no additional equipment available other than what is specified in the problem.

1. A married couple with each spouse of equal weight, together with two children, each weighing half that of one parent, wish to cross a river using a boat which can only carry the weight of one adult. Nobody in the family can swim. This is not a trick question.

**Can this be done?** Justify your answer.

2. Soldiers...retreating!

- (a) Three soldiers must cross a river. Two boys have a boat and are willing to help. Their small boat can hold either the two boys or one soldier. How many moves are necessary to get all across? What are the moves?

- (b) What if there are five soldiers (plus the two boys)?

- (c) During the Turkish stampede in Thrace, a small detachment found itself confronted by a wide and deep river. They discovered a boat with two rowing children. It was so small that it could hold only the two children, or one grown up person. How did the officer get himself and his 537 soldiers across the river and leave the two children in possession of their boat? And how many times need the boat to pass from shore to shore?
3. During a country ramble, Mr. and Mrs. Softleigh found themselves in a pretty little dilemma. They had to cross a stream in a small boat which was capable of carrying only 150 lbs. weight. But Mr. Softleigh and his wife each weighed exactly 150 lbs., and each of their sons weighed 75 lbs. And then there was the dog, who could not be induced on any terms to swim. On the principle of “ladies first,” they at once sent Mrs. Softleigh over; but this was a stupid oversight, because she had to come back again with the boat, so nothing was gained by that operation.
- How did they all succeed in getting across?**

4. Don, Sony, Angela, and John come to a river in the night. There is a narrow bridge, but it can only hold two people at a time. They have one torch and, because it's nighttime, the torch has to be used in order to cross the bridge. Don can cross the bridge in one minute, Sony in two minutes, Angela in five minutes, and poor old John (not our John, of course) in eight minutes. When two people cross the bridge together, they must move at the slower person's pace. The torch holds enough kerosene to last for just 15 minutes.

**Can they all get across the bridge before the lamp runs out?**

If so, how? If not, why not?

Write down ALL your attempts to solve this problem. If you're wrong, don't erase, but start over below your previous attempt.

5. Don, Sony, Angela, John, and Jeff come to a river in the night. There is a narrow bridge, but it can only hold two people at a time. They have one torch and, because it's nighttime, the torch has to be used in order to cross the bridge. John (with his skateboard) can cross the bridge in a whopping one minute, Jeff in three minutes, Angela in six minutes, Don in eight minutes, and Sony (exhausted from studying for finals) in twelve minutes. When two people cross the bridge together, they must move at the slower person's pace. The torch holds enough kerosene to last for just 30 minutes.

**Can they all get across the bridge before the lamp runs out?**

If so, how? If not, why not?

6. Once upon a time, a farmer went to the market and purchased a fox, a goose, and a bag of beans. On his way home, the farmer came to the bank of a river and rented a boat. But in crossing the river by boat, the farmer could carry only himself and a single one of his purchases - the fox, the goose, or the bag of the beans. If left alone, the fox would eat the goose, and the goose would eat the beans. The farmer's challenge was to carry himself and his purchases to the far bank of the river, leaving each purchase intact.

**Could he have succeeded?** Justify your answer.

7. Three missionaries and three cannibals must cross a river using a boat which can carry at most two people. But there's another catch: for both banks, if there are missionaries present on the bank, they cannot be outnumbered by cannibals. (If they were, the cannibals would eat the missionaries!) The boat cannot cross the river by itself with no people on board.

**Can the crossing be made?** Justify your answer.

8. A man lives on the island of Maui near a ferry dock. He has two children, a son on the nearby island of Lanai and a daughter on the nearby island of Molokai. To visit the son in Brooklyn, he takes a ferry from the left side of the dock; to visit the daughter, he takes a ferry on the right side of the same dock. Since he likes both of his children equally well, he simply takes the first ferry that comes along. In this way, he lets chance determine whom he visits. The young man reaches the boat dock at a random moment each Saturday afternoon. Lanai and Molokai ferries arrive at the dock equally often—every 10 minutes. When the ferry arrives, it picks up its passengers and leaves in a negligible amount of time. Yet, for some obscure reason, the man finds himself spending most of his time with his son on Lanai: in fact on the average he goes there 9 times out of 10. Can you think of a good reason why this happens?