Tiling figures with polyominoes

ORMC

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1 Intro to tilings

Problem 1.1. Can you cut a 10×10 board into 1×3 rectangles? In this and the following problems you can rotate the pieces as you want.

Problem 1.2. a) Can you cut a chessboard with squares a1 and h8 cut out into dominoes?

b) What about a1 and a8?

Problem 1.3. Can you cut a 10×9 board into 1×6 rectangles?

Problem 1.4. A black exterior cube $4 \times 4 \times 4$ is cut into 64 unit cubes. The faces of the cubes adjacent to the cuts are white. Can these be assembled into a $1 \times 8 \times 8$ parallelepiped with both 8×8 faces colored in a checkerboard pattern?

Problem 1.5. The cells of a 10×10 grid are colored in a checkerboard pattern. A 1×3 strip can completely cover three cells. What is the minimum number of non-overlapping and non-overhanging strips needed to cover all black cells?

Problem 1.6. A 10×10 grid is tiled with T-tetrominoes and L-tetrominoes (the pieces can be rotated and flipped).

a) Show that we need to use at least one L-tetromino. *Hint: use checkerboard coloring.*

b) What is the maximum number of L-tetrominoes that could have been used?

2 Olympiad problems

Problem 2.1 (Moscow 2020)**.** Ten cells have been cut out from a chessboard. It is known that among the cut-out cells, there are both black and white cells. What is the maximum number of two-cell rectangles that can still be guaranteed to be cut out from this board?

Problem 2.2 (Simplified from Peru 2012)**.** Diego wants to cover a 6×6 square with eighteen dominoes. Can he place 3 dominoes (without overlaps) so that the remaining part of the grid can be covered with the remaining dominoes in exactly one way?

Problem 2.3 (Tournament of Towns, 1994)**.** Consider an 8×8 square on a plane, divided into 1×1 cells. The square is covered with right isosceles triangles (two triangles cover one cell). There are 64 black and 64 white triangles. We consider "correct" coverings such that any two triangles sharing a side are of different colors. How many correct coverings are there?

Problem 2.4 (Tournament of Towns 2003)**.** Can a 2003×2003 board be tiled with 1×2 dominoes, which are allowed to be placed horizontally, and 1×3 rectangles, which are allowed to be placed vertically? (Two sides of the board are considered horizontal, and the other two are considered vertical.)

Problem 2.5 (Russia 2015)**.** A 100×100 square grid has 1950 dominoes (two-cell rectangles) cut out along the cell boundaries. Prove that it is possible to cut out a four-cell figure in the shape of a T (possibly rotated) from the remaining part of the grid. (If such a figure already exists among the remaining parts, it is considered that it can be cut out.)

Problem 2.6 (Ural Tournament of Young Mathematicians)**.** Rotating and flipping ascentums and descentums is not allowed. A grid figure was assembled from 2017 ascentums and n descentums. Could it be that this figure can be partitioned into 2015 ascentums and $n + 2$ descentums?