

Fun and Games 2

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As in competitions past, team up and compete against your classmates. The competition consists of three sections: (1) Puzzles to solve with your teammates; (2) Games in which you will play against other teams; and lastly, (3) an engineering challenge.

1 Puzzles (~40 minutes)

Present your solutions on a separate piece of paper. Correct solutions will be awarded the points indicated, incorrect solutions may still receive partial credit.

1.1 Analog Clock (3 pts)

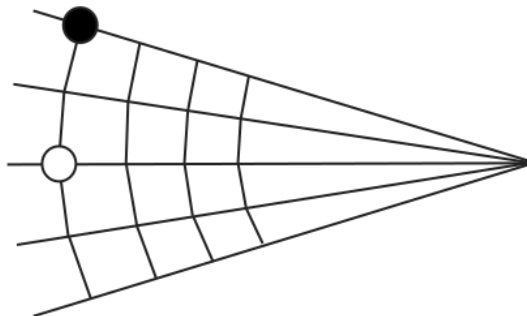
What is the measure of the smaller angle formed by the hands of the clock at 4:40?

1.2 Spider Pursuit (4 pts)

Shelob has infiltrated Charlotte's web. Can you help Charlotte catch Shelob?

1. A hub is an intersection between strands. There are 21 hubs as pictured below.
2. On each move, you may only move to an adjacent hub.
3. A move must be made on each turn.
4. Charlotte and Shelob move according to the same rules, taking turns.
5. Charlotte moves first.
6. Charlotte must catch Shelob by the end of her tenth move.

Charlotte starts at the white circle, Shelob starts at the black circle. Can you find the strategy that guarantees Charlotte can catch Shelob?



1.3 Around the World (5 pts)

Amelia Earheart wants to fly all the way around the equator without landing. Since her plane can only make it half way without refueling, she enlists the help of Bessie Coleman and Chuck Yeager. Amelia, Bessie, and Chuck each have identical airplanes that have the following peculiar properties:

1. Each plane travels 1 degree of longitude per minute using 1 kiloliter of fuel.
2. Each plane holds at most 180 kiloliters of fuel.
3. Any two planes can instantly transfer any amount of fuel in mid-air given that they are next to each other.
4. The planes can turn around instantaneously.

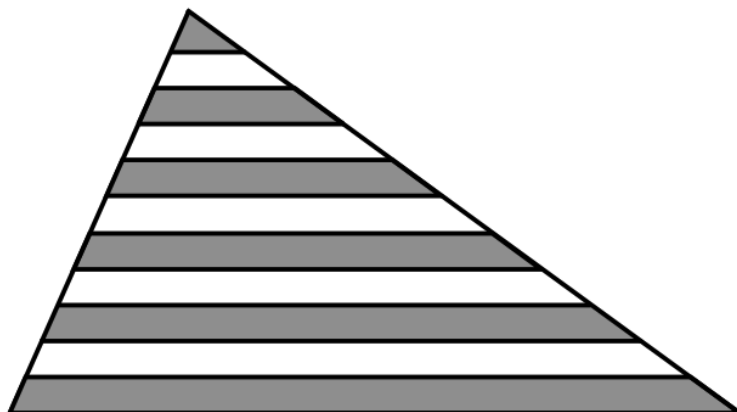
There is only airport from which all three planes may takeoff, land, and refuel. It happens to be on the prime meridian (0° longitude) and the equator (0° latitude). How can Amelia make it all the way around the world without either of her friends running out of fuel and crashing?

1.4 Ants in a Tube (3 pts)

Jack Kerouac and Jane Austen stand at each end of a thin, long tube. Jack sends 50 ants towards Jane, one after another. Jane sends 20 ants towards Jack. All ants travel in a straight line along the tube. Whenever two ants collide, they simply bounce back and start traveling in the opposite direction. How many ants reach Jack and how many reach Jane? How many ant collisions take place?

1.5 Striped Triangle (5 pts)

A triangle of area 770 cm^2 is divided into 11 regions of equal height by 10 lines that are all parallel to the base of the triangle. Starting from the top of the triangle, every other region is shaded, as shown. What is the total area of the shaded regions?



2 Games (~ 50 minutes)

Play two rounds of the following games, each team starting once. The winner of each round receives 5 points.

2.1 Hackenbush

1. There is a long horizontal line at the bottom of the picture known as a ground line. All line segments in the picture must be connected by some path to the ground line. Otherwise, they are illegal line segments and cannot be part of the picture.
2. Line segments are separated by dots which each represent a vertex. Multiple line segments can branch out from the same vertex. (Note: Line segments do not have to be straight, and can even curl back around to connect to the same vertex that it began from.)
3. On each player's turn, he or she chooses and erases one of the line segments. Then, if any line segments are no longer connected by some path to the ground line, they fall off of the board since they are now illegal, and will be erased.
4. On each player's turn, if there are no line segments left on the board, that player loses the game. Accordingly, the player to erase all remaining line segments on the board wins (just like taking away the last piece in nim).

2.2 Symbol Strike 3.0

- In addition to all the previously allowed symbols (0-9, +, -, ×, ÷), we shall introduce two new symbols: the decimal place (.) and exponentiation (^)
- Each team now has 13 spaces
- Each team makes the first move on their own paper
- Take turns switching papers, maximizing your own expression and minimizing your opponent's
- $05 = 5$ is allowed
- $1 + -5 = -4$ is allowed but $1 - +5$ is not
- $.5 = 0.5$ and $5. = 5$ are allowed
- Let $n > 0$, we shall define:¹
 - $n \div 0 = \infty$ and $-n \div 0 = n \div -0 = -\infty$
 - $\pm n^\infty = \pm\infty$ and $n^{-\infty} = 0$
 - $0^{\pm\infty} = 0$
 - $0 \div n^{-\infty} = 0$

¹These are all arbitrary definitions chosen for the game. Expressions such as $0 \div 0$ and 0^∞ are called *indeterminate* and are not definite real numbers. You will learn more about them in calculus.

3 Cantilever Challenge (~ 20 minutes)

On a desk, build a structure out of the provided materials. Your structure may not touch any vertical surfaces, only the horizontal surface of your desk. You will receive 1 point for every centimeter your structure extends past the edge of the surface.