LAMC Junior Circle **December 2, 2012**

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Warm-up

Problem 1 *Recover the following computations by placing the digits,* 0 *through* 9*, in the boxes below.*

a.

$$\begin{array}{r} \square\,\square \\ +\quad\ \square \\ \hline \square\,\square\ 8 \end{array}$$

b.

$$\begin{array}{r} \square\,\square \\ \times\ 5\ 2 \\ \hline \square\ 6 \\ +\ \square\,\square\ \ \\ \hline \square\ 7\ \square \end{array}$$

Problem 2 *Prove that out of any three integral numbers, you can always choose two with an even sum.*

Problem 3 *The awaken Sleeping Beauty throws a party for the seven dwarfs. All the dwarfs wear hats of different sizes.*

Sleeping Beauty and the seven dwarfs.[1]

[1] Illustration by Franz Jüttner (1865 – 1925).

The dwarfs leave the party in a very merry mood. The first to walk away takes the first hat he can put on his head, possibly of the size larger than his own. The second dwarf puts on a hat in a similar way, and so on. In the end, some dwarfs can't find a hat of the size they can put on their heads and leave without a hat. What is the maximal number of dwarfs that can leave without a hat?

Solve the same problem for 777 dwarfs instead of 7.

Problem 4 *Two players play the following game. Ten cards are put in a row on the table. The players take turns in taking the cards. A player can take either one card or two two cards next to each other off the table. The player who takes the last card off the table wins. Find a strategy to win the game.*

Problem 5 *The players are given 15 cards. They can take one, two, or three consecutive cards off the table at a time. Find the winning strategy. Can you generalize it to any number of cards bigger than 4?*

The average value

The *average value* is the typical value of a data set. Here is an example. For a week, Gregory was counting how many ice-creams he ate every day. The boy came up with the following table.

Monday	2
Tuesday	1
Wednesday	3
Thursday	2
Friday	1
Saturday	4
Sunday	3

He added up the numbers, divided by seven

$$\frac{2+1+3+2+1+4+3}{7} = \frac{16}{7} = 2\frac{2}{7}$$

and concluded that on average he eats two and two seventh of an ice-cream a day.

Problem 6 *Suppose that we only know that Gregory on average eats* $2\frac{2}{7}$ *of an ice-cream a day.*

- *Could it be that he eats no more than two ice-creams a day? Why or why not?*

- *Could it be that he eats more than two ice-creams every day of the week? Why or why not?*

- *If Gregory starts eating an ice-cream, he always finishes it off. Given this information, could it be that he eats more than two ice-creams every day of the week? Why or why not?*

- *Could it be that during one day of the week Gregory managed to eat 22 ice-creams? Why or why not?*

- *What is the minimal average value of ice-creams per day found over the period of one week that gives Gregory a chance to eat 22 ice-creams in one day?*

You will need to cooperate with other students at your table to solve the following.

Problem 7 *What is the average number of languages the students at your table speak?*

Problem 8 *Suppose that you only know the average number of languages the students at your table speak. Could it be that the only language the students at your table speak is English? Why or why not?*

Problem 9 *Knowing only the average number of languages the students at your table speak, could it be that one of the students speaks 5 languages? Why or why not?*

Discuss the following with other students and the instructor at your table.

Problem 10 *In the year 2010, the average household income in the US was $67,530. What do you think this means and what is this number good for?*

Problem 11 *There are 5 brothers in the family. Their average age is 9 years. Six years ago the average age of the siblings was 4 years. How old is the youngest brother?*