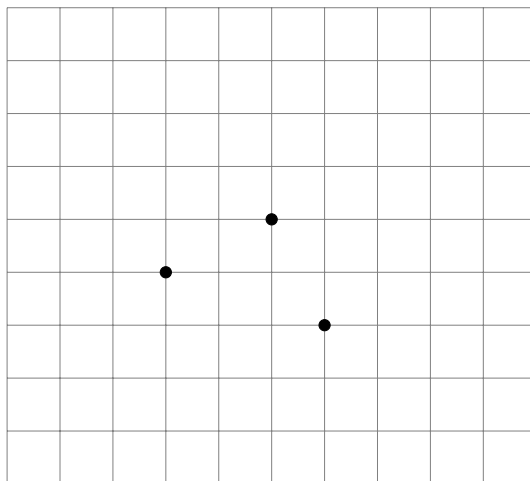## Vectors and Physics 5, Review Session

Oleg Gleizer

prof1140g@math.ucla.edu

Problem 1 *Three points are marked on the grid below. Draw the fourth point so that the four points and the lines connecting them form a parallelogram. Find all the possible solutions!*

Problem 2 *Use a compass and a ruler to divide the segment below into three parts of equal length.*
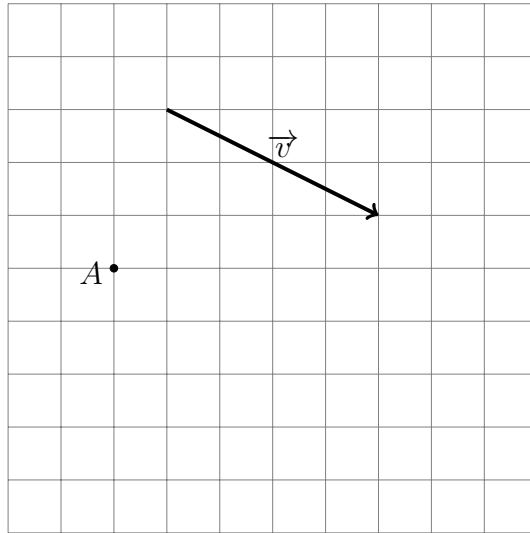
Problem 3 *Use a compass and a ruler to construct the sum $\vec{v} + \vec{w}$ of the vectors $\vec{v}$ and $\vec{w}$ given below.*

$\vec{v}$

$\vec{w}$
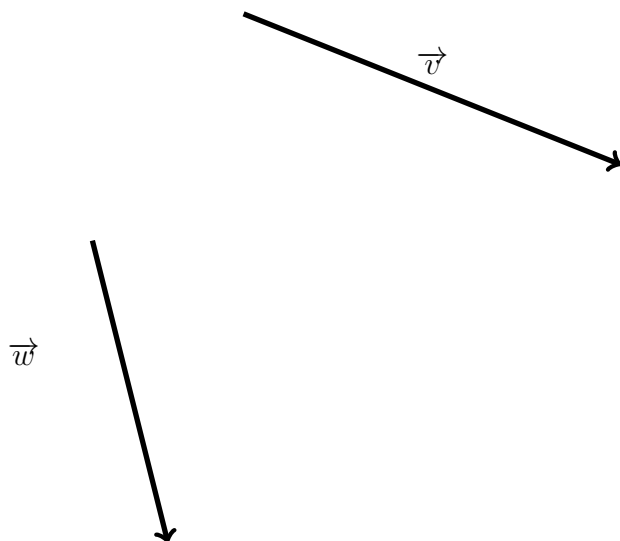
Problem 4 *Prove that addition of vectors is commutative.*

$$\vec{v} + \vec{w} = \vec{w} + \vec{v} \tag{1}$$

Problem 5 *For the given vector $\vec{v}$ and point A, construct the vector $\vec{w} = -1.5\,\vec{v}$ having A as its terminal point on the graph paper below.*

Problem 6 *For the given vectors $\vec{v}$ and $\vec{w}$, construct the vector $\frac{2}{3}\vec{w} - .75\,\vec{v}$ on the graph paper below.*

Problem 7 *Cut the two squares below into five parts such that one can make one square out of the parts, using all the parts and without overlapping. Use a compass and a ruler to draw all the necessary lines with maximal precision!*
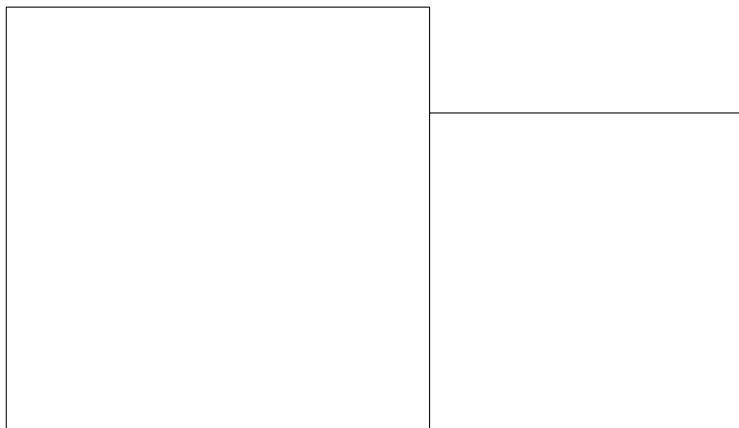
Problem 8 *Given the segment of length a, use a compass and ruler to construct a segment of length $\sqrt{2}a$ in the space below.*

$$a$$

$$\overline{\hspace{6cm}}$$

Problem 9 *Use a compass and ruler to construct a segment of length $\sqrt{5}a$ in the space below where a is the length of the segment from Problem 8.*

Problem 10 *Use a compass and a ruler to construct the vector* $\overrightarrow{w} = \sqrt{5}\overrightarrow{v}$ *for the vector* $\overrightarrow{v}$ *given below such that point C is its initial point.*

$\overrightarrow{v}$
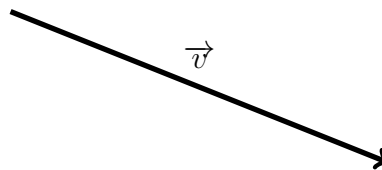
$\bullet$
C

**Problem 11** *Give the definition of a rational number.*

Problem 12 *Prove that $\sqrt{5}$ is not rational.*

Problem 13 *A man is crossing a river in a boat. The speed of the boat is five metres per second. The speed of the water in the river is two metres per second. In what direction should the man steer the boat, if he wants the vessel to move perpendicular to the banks? Please use a compass and a ruler to construct the velocity vector.*

$2\frac{m}{s}$
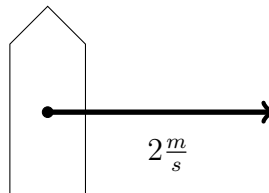
The width of the river is $60\sqrt{21}$ meters. How long would it take the man to cross the river?

Problem 14 *There are no forces acting on the object A below. The current velocity of the body, in metres per second, is represented by the vector $\vec{v}$. Draw the position of the object three seconds later.*
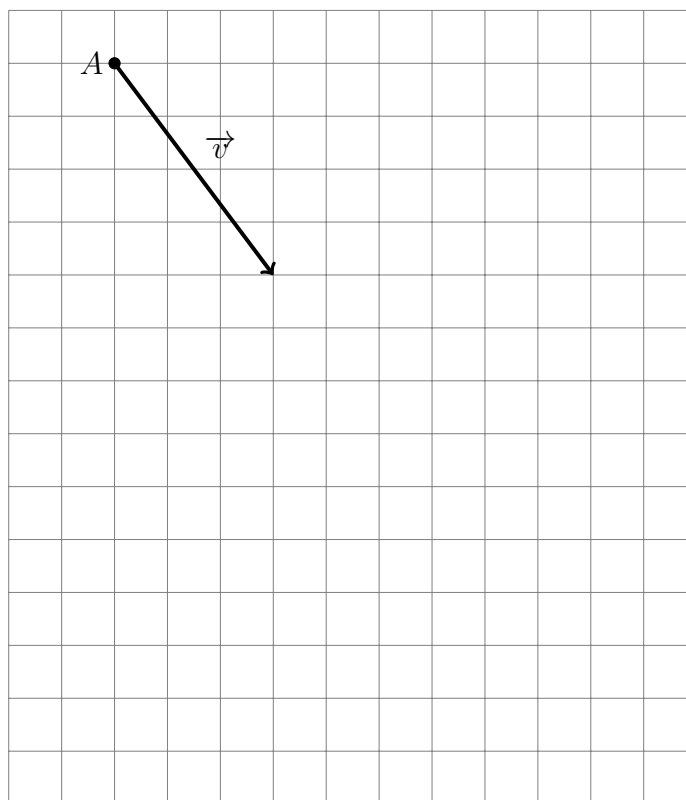

The sides of the grid squares on the picture above are ten metres long. Find the speed of the motion. (Please continue to the next page.)

$$v =$$

What distance would the object cover in a minute?

$$d =$$

Problem 15 *Moving with a constant acceleration, a body changes its position from point A to point B in one second. The velocities of the motion, in metres per second, are represented by the vectors $\vec{v_0}$ and $\vec{v_1}$. Find the acceleration vector.*
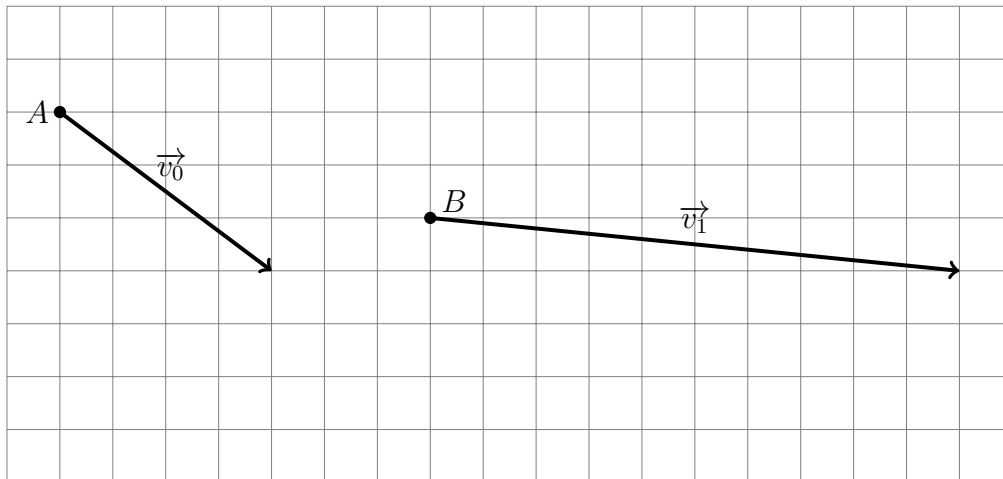
Problem 16 *The General Electric F110-GE-129 turbofan engine powering the F-16 fighter jet generates 29,588 lbf (131.6 kN) of thrust.*

F-16

The aircraft, armed and loaded with fuel, weighs 26,500 lbs (12,000 kg). The free-fall acceleration near the Eath's surface is $9.8\ m/s^2$ ($32.2'/s^2$). Is a fully loaded F-16 capable of ascending vertically? Why or why not?

Problem 17 *Use vector algebra to prove that all the three medians of a triangle in the Euclidean plane intersect at one point that splits each of the medians in the ratio 2:1 counting from the vertices.*
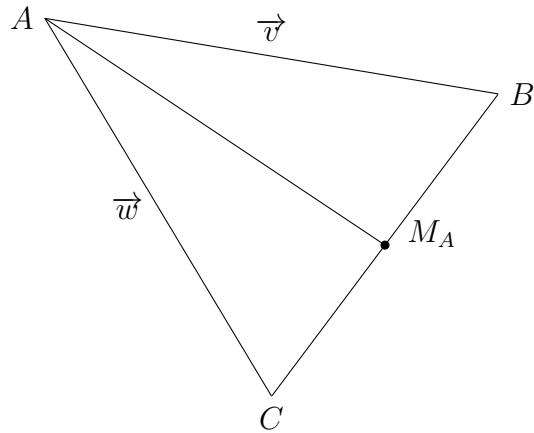
Problem 18 *Draw the scheme of a pulley with the advantage two. Explain how it works.*