Intermediate 1 **April 14, 2024**

Financial Literacy 1: Investment, Interest, Time Value of Money, and Future Value

1 Warm-up

Problem 1.1. *You have two investments (A and B), each requiring a $1,000 payment upfront and each expected to return $100 interest over a two-year holding period. Which one is more desirable given the payment schedule below?*

Investment	Year 1 (in dollars)	Year 2 (in dollars)
A	$50	$50
B	$0	$100

Table 1: Two Investments

2 Investment

An **investment** can be described as an opportunity to which we can put our money to **create more money in the future**, or, at least, **preserve its value**.

Why do we invest in the beginning? Besides **the desire to be rich**, there are numerous reasons people choose to invest their income or savings:

1) An extra source of passive income that helps us make large purchases like getting our first car, buy our dream house, and pay for university tuition. As we'll learn later in the chapter, a fundamental form of investing is putting our money to bank, which "rents" our money and pays us for the time we keep our money there, as we call it **"interest."** Interest can be a very powerful tool over time, helping us collect large amounts of money faster.

2) Preserving the value of our income and savings. As we are going to explain in future chapters, prices of goods and services, ranging from apples and bananas to house prices and electricity bills, tend to increase over time. As prices increase, the amount of goods we can buy with our salary decreases. To prevent this from happening, investment is a good way to match the price increases with some extra money we generate.

3) Preparing for retirement. Most of us will start working right after graduation from college and continue for many years, perhaps retiring at 60. After retirement, we won't be getting a salary or income anymore. Therefore, investing is very important to both earn extra income until we retire, which we can spend afterwards, and also perhaps continue the grow our savings even after retirement.

The rewards of investments come in two natures: **income** and/or **increase in value**. Let's say we buy a house for \$ 1 million in Los Angeles and rent it out to someone. The rental income we earn every month falls under **income** category of an investment. Also, let's say that housing prices in Los Angeles increase in general over time. We find out that we can sell our house for \$1.5 million after 2 years since we bought this house. This represents an **increase in value**, another type of investment reward.

Problem 2.1. *What are some other goals we can use investing to achieve?*

Problem 2.2. *What are some investing ways you have heard about before? Where can people put their money to earn more money?*

Problem 2.3. *Is investing always a guaranteed way to get rich? Can an investment go wrong?*

3 Interest

Interest is the monetary charge of borrowing money, typically expressed as an annual percentage rate.

In simpler terms, interest is like a rent. When we put our money in the bank, the bank uses our money in its own operations, even though we don't notice a change on our account balance digitally. Therefore, as a compensation for using and taking care of our money, the bank provides us "interest."

On the other side, if we borrow money from a bank, by taking a loan or using credit card, we are the ones who "rent" money from bank. Therefore, as a compensation of using and taking care of bank's money for some time, we pay back our loan with "interest."

For the sake of simplicity, we will focus more on the first part, where we put money on bank and "lend" them our money, as this falls under "investing." We won't be involved with borrowing money from a bank, or taking a loan, for now.

Summary: Think of interest as a "rent" paid by the borrower for use of the lender's money.

Example 1. *The bank calculates interest as a percentage of the total amount in a bank account. For example, if the bank pays 1**% interest** annually and there is **$500** in your account, then you will earn **$5** in interest over a year.*

Simple Interest: Simple interest is calculated by multiplying principal money by the interest rate and then by the term of a loan.

$$Interest(Simple) = P * r * n \tag{1}$$

P is principal, r is the interest rate, and n is term or period.

Example 2. *Say you have \$100 investment for 2 years, paying 5% annual interest. Since you only put interest on your initial investment, your 1st year interest payment and 2nd year interest payment will be same. For **1st year**, \$100 * 5% = **\$5**, and, for **2nd year**, \$100 * 5% = **\$5** as well. In total we will get \$10 interest payment. (\$100*2*5% = \$10)*

Example 3. *Let's say that a student obtains a simple interest loan to pay for one year of college tuition. The loan amount is \$20,000. The annual interest rate on the loan is 5%. The term of the loan is three years. What is the amount of interest paid and total amount?*

$$\$20000 * 5\% * 3 = \$3000 \tag{2}$$

Total Amount is principal and interest, \$20,000 + \$3,000 = \$23,000.

Problem 3.1. *If you invest \$ 500 at a 5% annual interest rate, how much interest will you have earned after 3 years?*

Problem 3.2. *You borrow \$2,000 at a 6% annual interest rate. How much interest will you owe after 4 years?*

Problem 3.3. *If \$800 is deposited into a savings account with an annual interest rate of 3%, how much total interest will be earned after 2 years?*

Compound Interest: Compound interest is interest that applies not only to initial investment, but also the accumulated interest from previous periods. In other words, compound interest is earning interest on interest.

Example 4. *Say you have \$ 100 investment for 2 years, paying 5% compound interest annually. For **1st year** your interest payment will be same with simple interest (\$100 * 5% = \$5.) For **2nd year**, however, your interest will be calculated on not only initial investment, but instead, initial investment + accumulated interest payment. (\$105 * 5% = \$5.25). In total, we will have **\$10.25** interest payment.*

Problem 3.4. *You have P money to invest for a year at the rate r. How much would you have at the end of the year?*

Problem 3.5. *A) You have P money to invest for two years at the rate r compounded annually. How much would you have at the end of the two-year period? B) what would be the interest compounded after one year?*

Hence, we can define compound interest as: To see what your investment is worth after n years, the formula is:

$$FinalAmount = P * (1 + r)^n \tag{3}$$

P is principal, r is the interest rate, and n is term/period.

Difference between Simple vs Compound Interest: Simple interest is better for borrowers because it doesn't account for compound interest. On the other hand, compound interest is a key to building wealth for investors. Therefore, for the sake of our module's focus on investment, we will work with compound investment from now and onwards.

Problem 3.6. *If you invest $1,000 at an annual interest rate of 4% compounded annually, how much will the investment be worth after 5 years?*

Compounding Periods: Interest may be compounded more frequently than once per year. Many bank accounts pay interest monthly. If that is the case, simply divide the annual interest rate to the period that is charged (if monthly, divide by 12) and compound by the equivalent period amount (if monthly, 12 * year).

Problem 3.7. *You have P money to invest for a year at the rate r compounded monthly. How much would you have at the end of the year?*

Problem 3.8. *You have P money to invest for n years at the rate r compounded monthly. How much would you have at the end of the n-year period?*

Problem 3.9. *You have P money to invest for n years at the rate r compounded k times a year. How much would you have at the end of the n-year period?*

Hence, over general formula for **compound interest** will be:

$$A = P(1 + \frac{r}{n})^{nt} \tag{4}$$

Where, ***A*** is the amount of money accumulated after n years, ***P*** is the principal amount (the initial sum of money), ***r*** is the annual interest rate, ***n*** is the number of times that interest is compounded per unit t, and **t** is the time the money is invested or borrowed for, in years.

Problem 3.10. *A sum of \$2,500 is deposited into a savings account with an annual interest rate of 3% compounded quarterly. What will be the total amount after 3 years?*

Problem 3.11. *You invest \$5,000 in a compound interest account with an annual rate of 5% compounded monthly. How much will your investment grow to after 10 years?*

Problem 3.12. *With an initial deposit of \$10,000 at an annual interest rate of 2.5% compounded monthly, what is the total amount after 1 year?*

4 Time Value of Money:

If you were offered \$100 today or \$100 a year from now, which would be the better option and why?

Most of the time, accepting \$100 today would be the correct answer. Three main reasons for this are gathered under **Time Value of Money Theory**. **First**, a dollar can be invested and earn interest over time, giving it potential earning power. **Second**, money is subject to inflation (rise in prices of goods and services over time), eating away at the spending power of the currency over time, making it worth a lesser amount in the future. **Finally**, there is always the risk of not receiving the dollar in the future, whereas if you hold the dollar now, you eliminate that risk. (Investopedia)

Time Value of Money is closely connected to Future Value.

5 Future Value:

Future Value is the amount that an investment will be worth at a future date if it is invested at a compounded interest rate.

Future Value for a Single Period is:

$$FV = C(1 + r) \tag{5}$$

Where, FV is future value, C is initial investment, and (1+r) is interest factor)

Future Value for a Multiple Periods are:

$$FV = C(1 + r)^t \tag{6}$$

Problem 5.1. *What is the future value of \$1000 at a 3% interest rate in 2 years?*

6 Chapter Problems:

Problem 6.1. *Alex deposits 1000 into a bank account that pays a compound interest rate of 4%, with interest credited at the end of each year. Determine the amount in Alex's account just after interest is credited at the end of the 1st, 2nd, and 3rd years.*

Problem 6.2. *1000 is invested. Find accumulated value of investment 5 years after it is made for each of the following rates: a) 5% annual simple interest rate b) 5% compound interest compounded annually c) 5% compound interest compounded quarterly d) 5% compound interest compounded monthly d) 5% compound interest compounded daily*

Problem 6.3. *At an annual effective interest rate of 12%, calculate the number of years (including fractions) it will take for an investment of 1000 to accumulate to 3000 (compound). (b) Repeat part (a) using the assumption that for fractions of a year, simple interest is applied. (c) Repeat part (a) using an monthly interest rate of 1% (compound). (d) Suppose that an investment of 1000 accumulated to 3000 in exactly 10 years at compound interest i. Calculate i. (e) Repeat part (d) using a monthly rate of interest j. Calculate j.*

Problem 6.4. *Unit values in a mutual fund have experienced annual growth rates of 10%, 16%, -7%, 4%, and 32% in the past five years. The fund manager suggests the fund can advertise an average annual growth of 1% over the past five years. What si the actual average annual compound growth rate over the past five years?*

Problem 6.5. *Bob puts 10,000 into a bank account that has monthly compounding with interest credited at the end of each month. The monthly interest rate is 1% for the first 3 months of the account and after that the monthly interest rate is .75%. Find the balance in Bob's account at the end of 12 months just after interest has been credit- ed. Find the average compound monthly interest rate on Bob's ac- count for the 12 month period.*

Problem 6.6. *Carl puts 10,000 into a bank account that pays an annual effective interest rate of %4 for ten years, with interest credited at the end of each year. If a withdrawal is made during the first five and one- half years, a penalty of 5% of the withdrawal is made. Carl withdraws K at the end of each of years 4, ,5 6 and .7 The balance ni the account ta the end of year 10 is 10,000. Calculate K.*

Problem 6.7. *A city population grows 5% every year. How long would it take to triple?*

Problem 6.8. *My dream car costs $10,000. I have $3000 in my bank account. I have the option to invest my money at an investment account at 5% compound rate annually. How many years do I need to wait to be able to buy my car?*

Problem 6.9. *You want to buy a $15,000 motorcycle in 5 years by investing an amount in an account that offers a 6% annual compound interest rate. How much money do you need to invest initially?*

Problem 6.10. *You have \$5,000 that you want to grow to \$8,000 in 10 years. What annual compound interest rate do you need to achieve this goal?*

Problem 6.11. *You invest \$4,000 in an account with a 4% annual compound interest rate. How long will it take for your investment to triple?*