LAMC Junior Circle **November 18, 2012**

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Warm-up

Problem 1 *A clock shows the time of 9:20. What is the angle between the hour hand and the minute hand?*

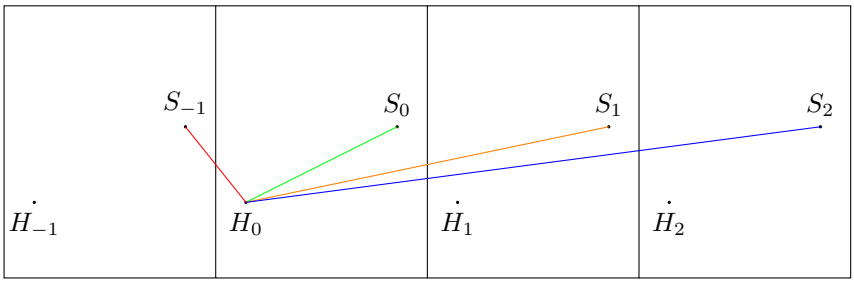
Problem 2 *Put the digits* 0 *through* 9 *in the boxes in such a way that each digit is used once and the product of the four numbers below is the largest possible.*

Geodesic lines on a cylinder

Problem 3 *What is a geodesic line on a surface?*

The purpose of the following construction is to show that there are infinitely many geodesic lines connecting a pair of points in general position on a cylinder.

Imagine that we have an infinite roll of paper covered with identical copies of a cylinder. Imagine further that we draw all the possible straight lines connecting the points H_0 and S_i, $i = \ldots -2, -1, 0, 1, 2 \ldots$ and, using a compass and a ruler, pull them back to the original cylinder, cut it out and glue the opposite sides of the cut together. We will end up with infinitely many different helical lines connecting the points H_0 and S_0.



Problem 4 *Implement the above idea for the sheets* -1, 0, 1 *and* 2. *Use different colors to draw the lines.*

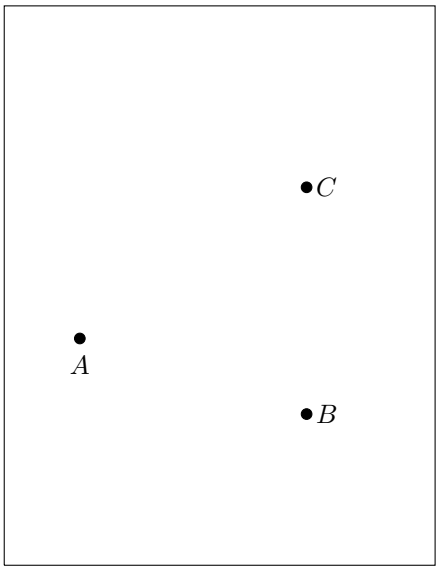

S
H


S
H

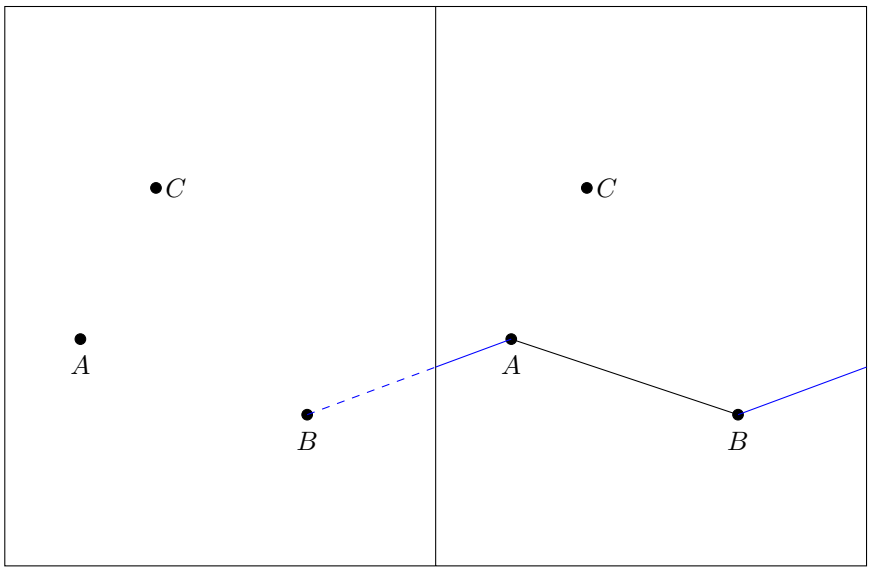

S
H


S
H

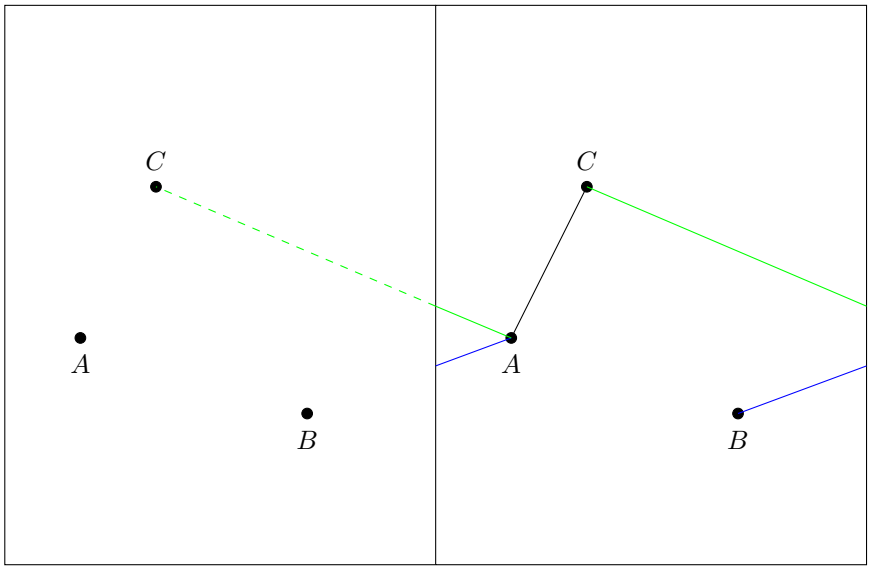
Example 1 *Draw a triangle having the below vertices on a cylinder.*



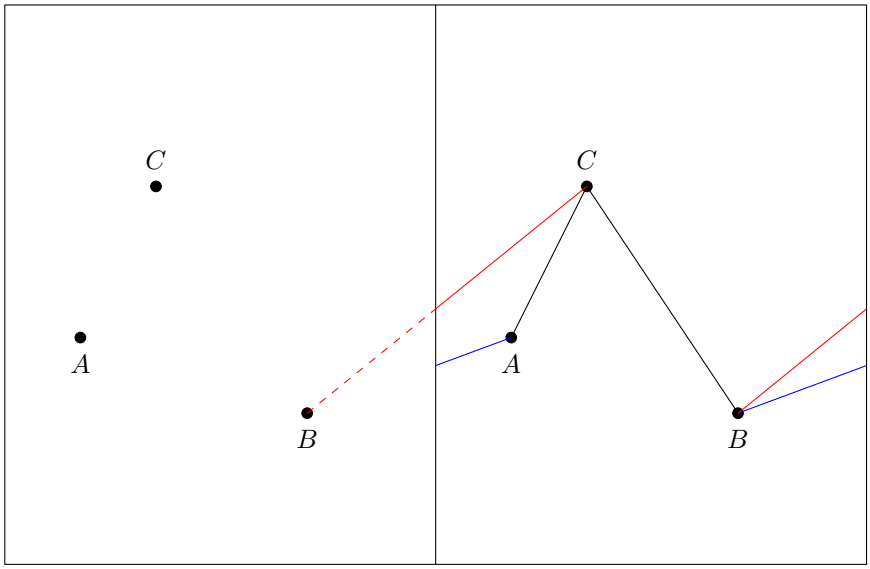
There are two candidates for the shortest line connecting A and B, one crossing the cut, the other avoiding it.



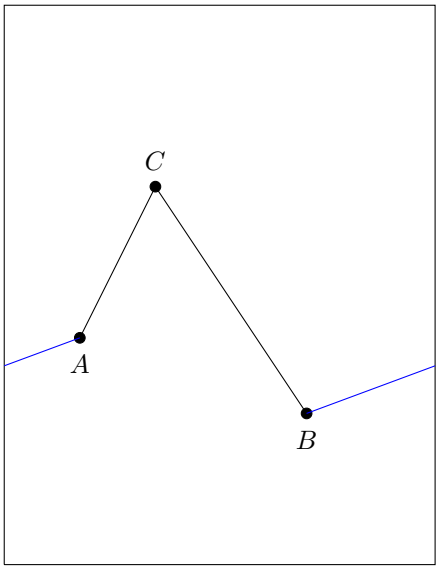
It is not visually clear which line is shorter, the blue or black one. A compass measurement shows that it's the blue line we are looking for. It is visually clear that the shortest path connecting A and C doesn't cross the cut.



It is more or less clear that the red line connecting B and C is longer than the black one.

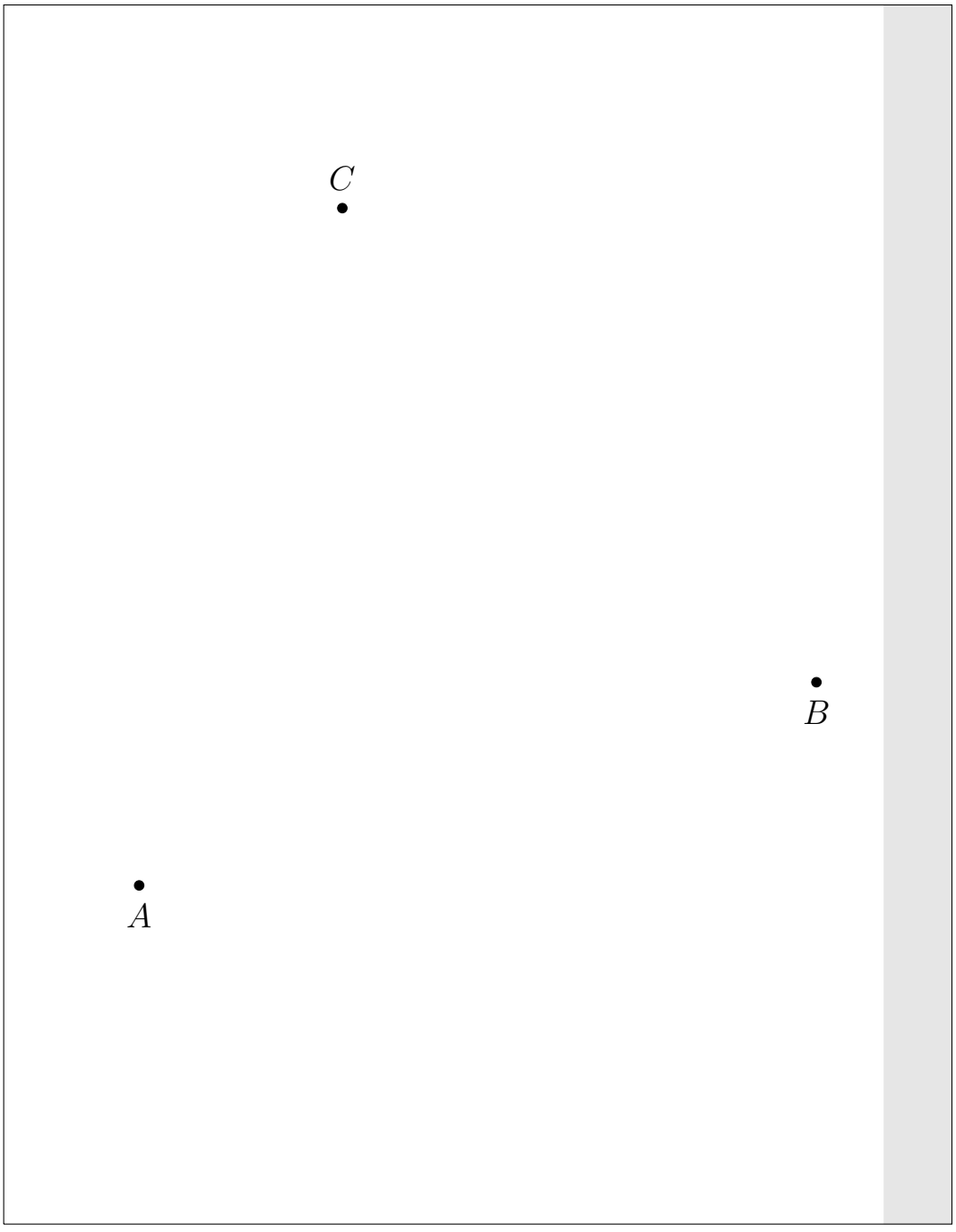


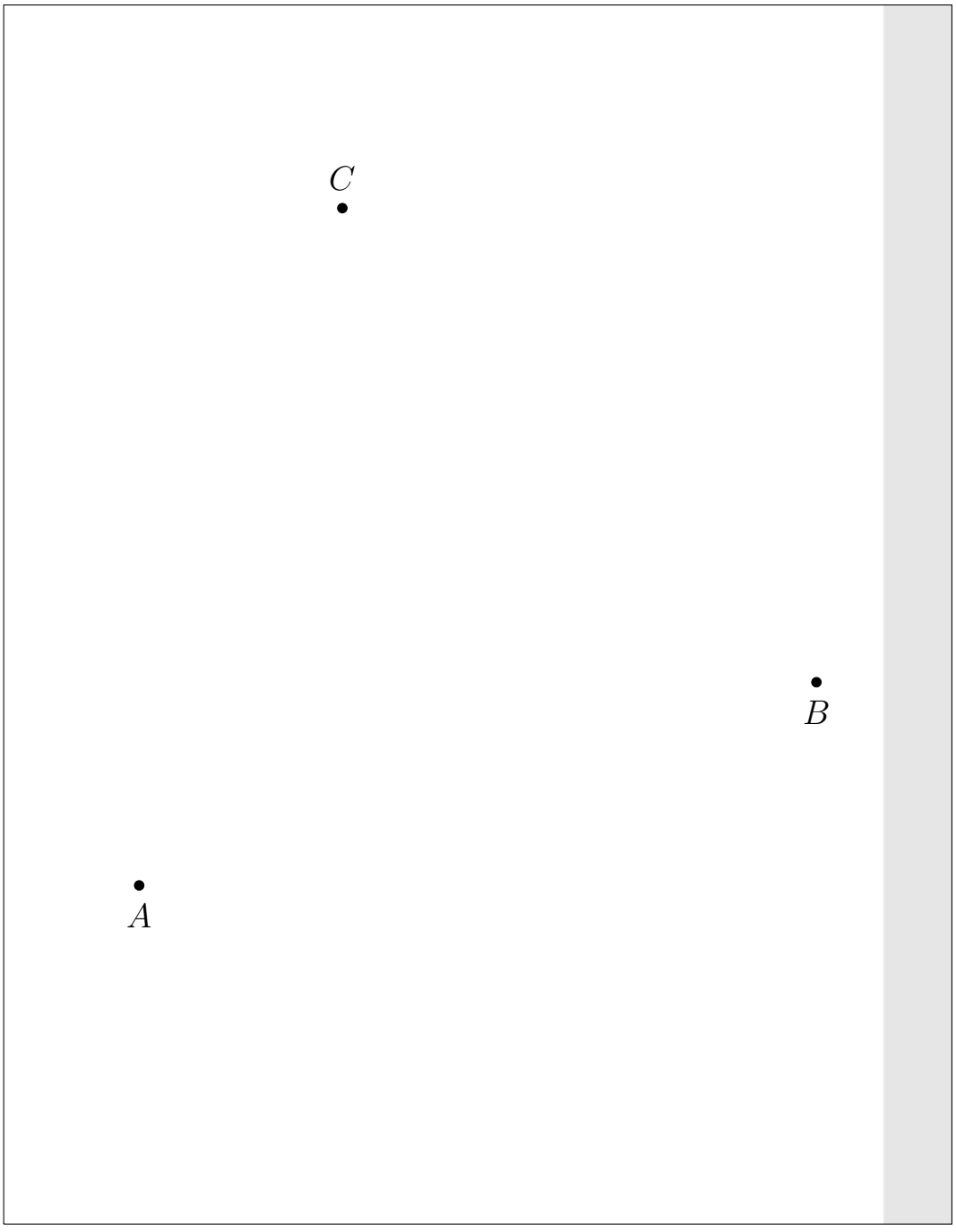
Double-checking with a compass confirms that there is no mistake. Finally, here comes the triangle.



It doesn't look like a triangle on a flat paper sheet, but becomes visually recognizable when we glue the sheet into a cylinder.

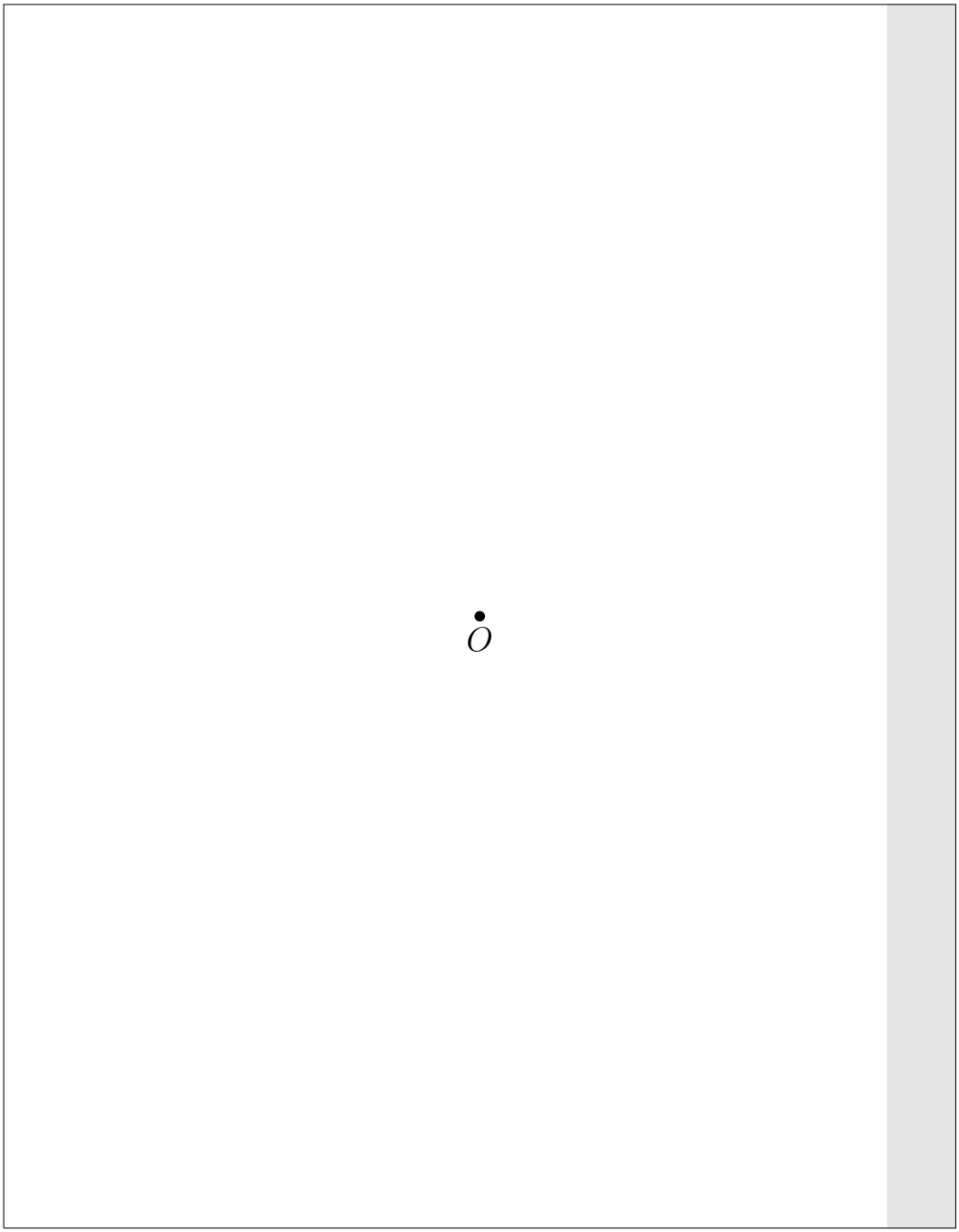
Problem 5 *Use the shortest possible paths connecting the points to construct the triangle with the given vertices on the cylinder below. As the first step of your construction, glue two copies of the cylinder together as above. Cut out and glue the main cylinder together at the final step.*

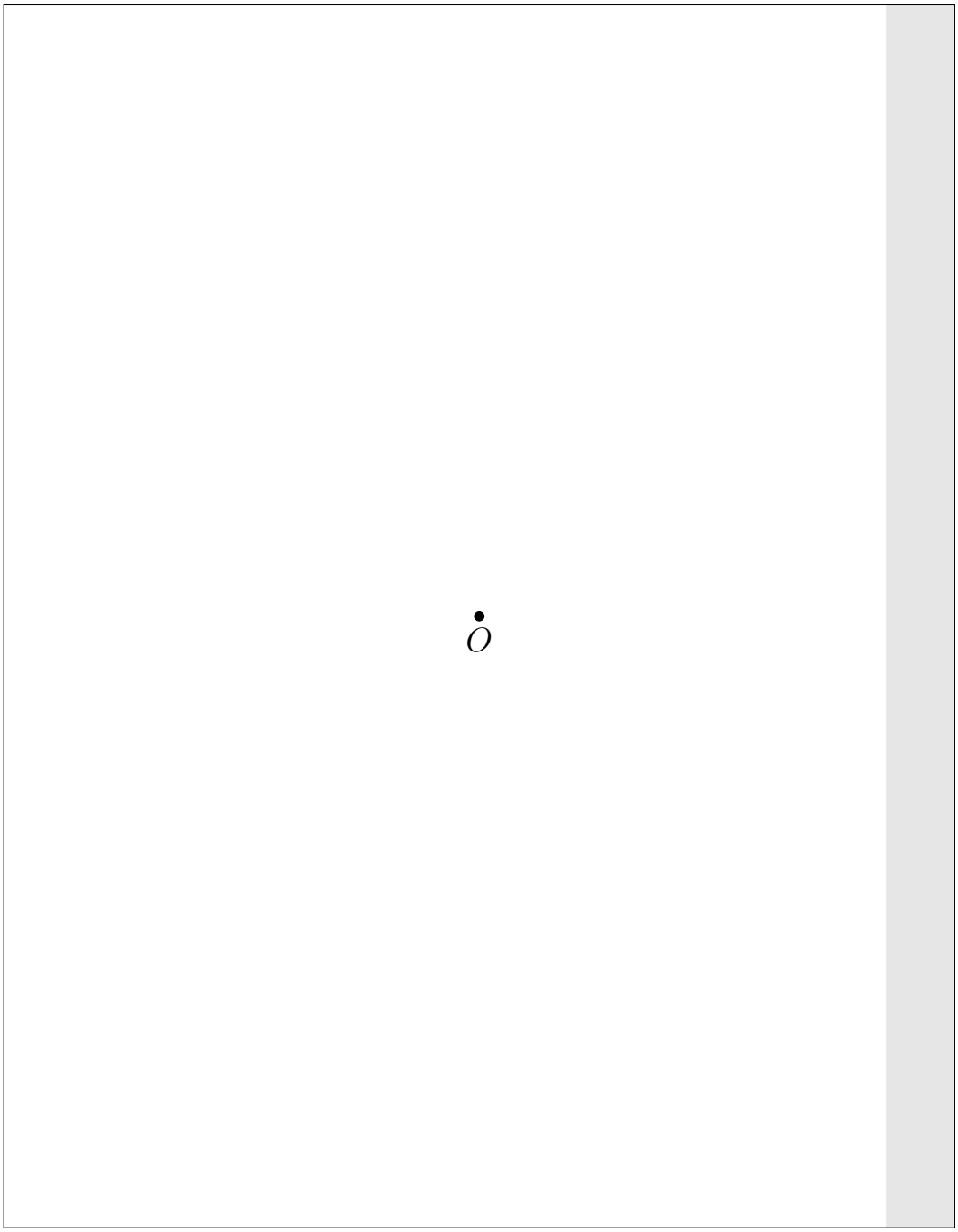

C
B
A

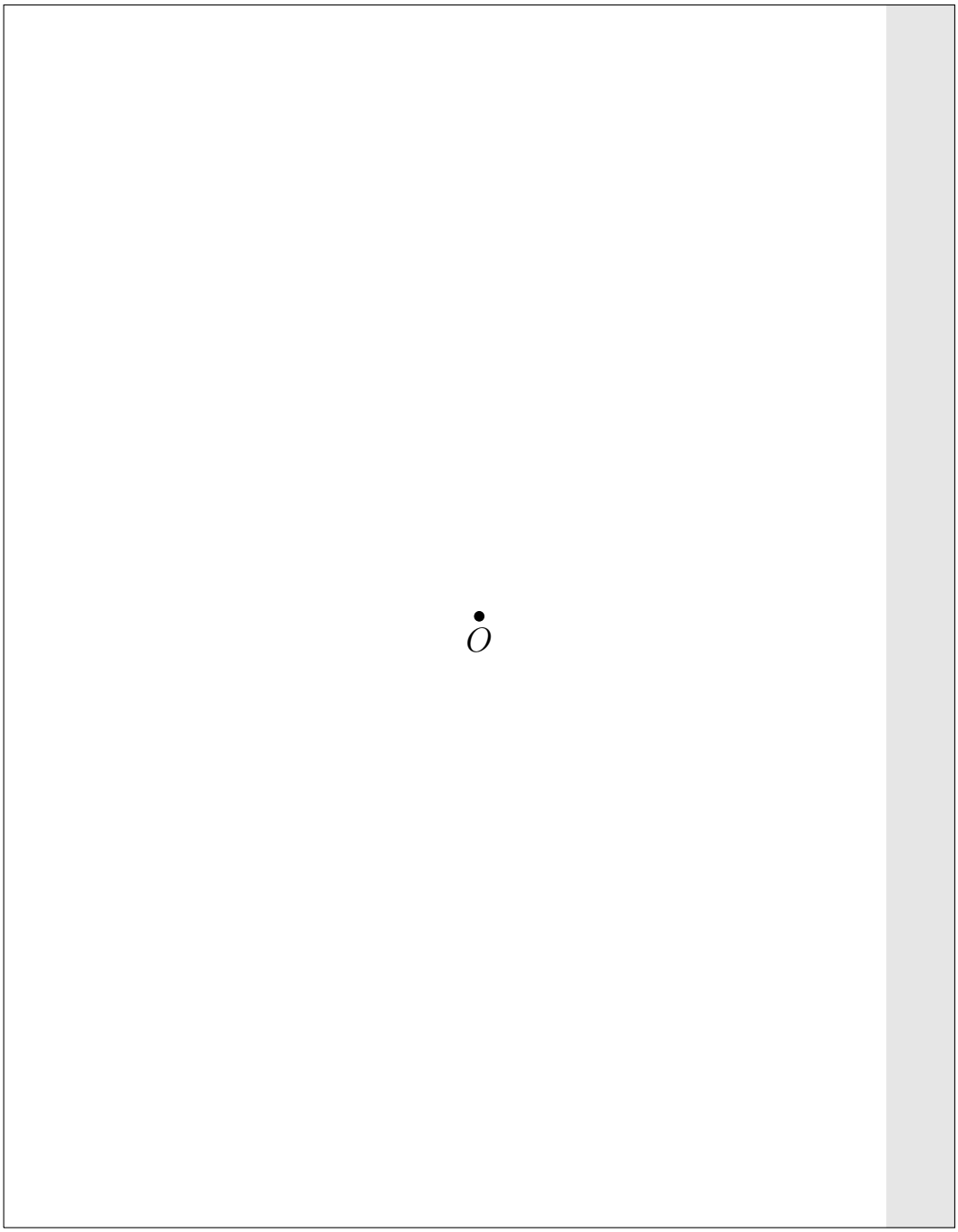

C
B
A

Similarly to the Euclidean plane, let us define a *circumference* on a cylinder as the set of all the points on the surface having the given distance, the *radius*, from the given point, the *center*.

Problem 6 *Use the three copies of the cylinder below to construct two circumferences, both centered at O, on the main cylinder. The radius of the first circumference is $1''$ (one inch), the radius of the second is $3''$. Cut out the main cylinder and glue it together at the last step.*


$\dot{O}$


$\dot{O}$


$\dot{O}$

If you are finished doing all the above, but still have time ...

Problem 7 *A farmer owns sheep and chickens. In total, the animals have 36 heads and 100 legs. How many sheep has the farmer?*

Problem 8 *There are 5 brothers in a family. Each of them has 1 sister. How many girls are there in the family?*

Problem 9 *Invent a question such that an honest answer to it is always "No."*

Problem 10 *A teacher made a one sentence statement that was true. A student repeated the statement without any change, but this time it was false. What was the statement?*