

Individual Problem Solving I

Math Circle (Intermediate)

November 4, 2012

Directions: This is designed as a check-in for you, your parents, and your instructors to examine how well everyone is doing this quarter. Solve the following problems in the space provided. Show ALL work! An answer without justification will receive no credit. Read all the problems carefully for any additional instructions. Underline your first name on this paper and you will receive a prize. If you don't understand what a question is asking, raise your hand; otherwise, try your best. Good luck!!

1. How many distinct ways are there to arrange the letters in the word "HALLOWEEN"?
2. How many "words" of length 2 can be formed using the letters in the word "HALLOWEEN"? Repeating letters is allowed.
[Do *not* attempt to solve this problem by writing out all the possibilities.]

3. **Prove or disprove:**

$$\binom{n}{k} = \binom{n}{n-k}$$

Bonus: Do it in two different ways: with math, and with an explanation.

4. Suppose there are 100 students enrolled in the Math Circle program.

Prove or disprove: There must be two people who were born in the same month AND on the same day of the week.

Show your work, and use complete sentences in any explanations.

5. Suppose ten students solved a total of 35 problems in a math olympiad. Each problem was solved by exactly one student. There are at least two students who solved exactly one problem, and at least two students who solved exactly three problems.

Prove or disprove: There must be at least one student who solved at least five problems.

A complete answer will identify the pigeons and the holes, and how many of each there are, and will use complete sentences in any explanations.

Class Rules:

6. Name three things you are expected to bring to class every day.
7. If you see a problem on the weekly handout that you don't know how to solve, what should you do?
- (a) stand up and walk over to the assistant at your table to ask them.
 - (b) ask your assistant; if they are helping someone else, ask the assistant at the next table over.
 - (c) write down as much of a solution as you can; if you're still stuck, ask the assistant at your table.
 - (d) skip it and move onto the next question.
 - (e) (b) or (c)

8. When and where are you allowed to eat a snack?
9. Suppose the person sitting across from you is talking about last week's baseball game. They begin talking to you about it. You're in the middle of problem 12 on your worksheet. What should you do? **Circle up to two answers.**
- (a) talk to them about the game. It's okay to just finish the handout at home.
 - (b) kindly let them know you are working and encourage them to do the same.
 - (c) notify the assistant at your table if you don't feel comfortable having the conversation in (b).
 - (d) tell them to shut up because you are trying to work.
10. Jeff has called on someone to show the class their solution to a problem. You've already solved that problem. What should you do?
- (a) since you've already solved the problem, it's okay if you whisper to your neighbor, but only if it's quiet.
 - (b) since you've already solved the problem, it's okay if you read your book.
 - (c) since you've already solved the problem, it's okay if you aren't listening, as long as you're working on other problems on the handout.
 - (d) even though you've already solved the problem, listen to them because they might have solved it a different way, and because your peers deserve respect.¹

¹Some problems are taken from:
D. Fomin, S. Genkin, I. Itenberg "Mathematical Circles (Russian Experience)"