AMC 8 Training: Triangles

Sahana Sri and Phoebe Xu

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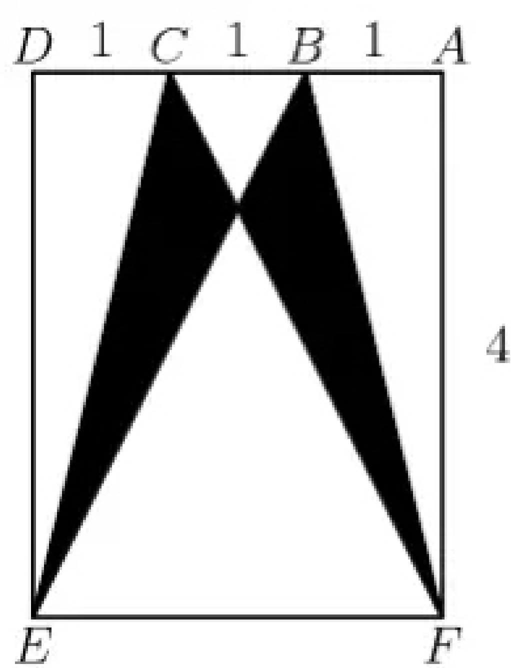
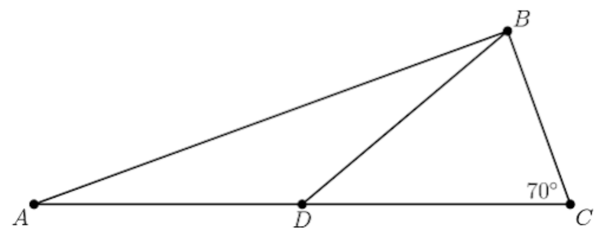
1 Warm Up

Rectangle $DEFA$ below is a $3x4$ rectangle with $DC = CB = BA$. What is the area of the "bat wings" (shaded area)?



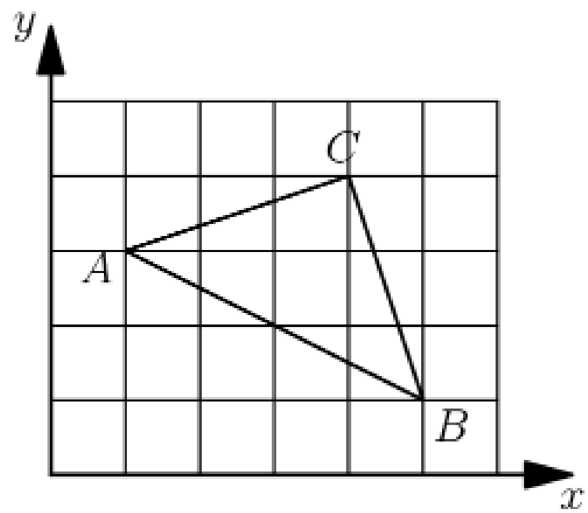
A)2 B) 2 $\frac{1}{2}$ C) 3 D) 3 $\frac{1}{2}$ E) 5

In triangle ABC, D is a point on side AC such that $BD = DC$ and angle BCD measures 70. What is the degree measure of angle ADB?



(A) 100 (B) 120 (C) 135 (D) 140 (E) 150

A triangle with vertices as $A = (1, 3)$, $B = (5, 1)$, and $C = (4, 4)$ is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle?



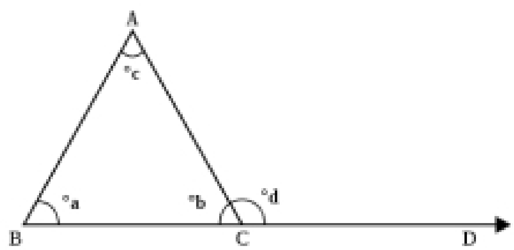
(A) B) 1/5 C) 1/4 D) 1/3 E) 1/2

2 Basics

Though you will probably not see a problem that utilizes only the following formulae, they are often helpful when breaking down larger, more complex geometry problems.

Area= 1/2 base x height
Perimeter: $s_1 + s_2 + s_3$
Angles: $a_1 + a_2 + a_3 = 180$

Find d in terms of a, b, and c for the triangle below.



Heron's formula can be used in place of the commonly known area formula. For Heron's formula, we need all three side lengths.

The semiperimeter is easy to calculate: it is half of the perimeter, or $P/2$.

Heron's formula is as follows:

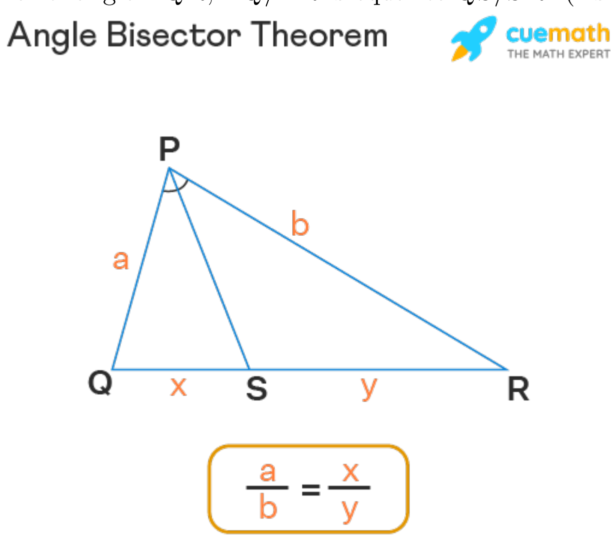
$A = \sqrt{s(s-a)(s-b)(s-c)}$

where s is the semiperimeter and a, b, and c are the side lengths.

Find the semiperimeter s for a triangle with side lengths 3, 4, and 5. Then find the area.

What is the area of a triangle with side lengths 13, 14, and 15?

Another useful formula is the Triangle Bisector Theorem, which states that for triangle PQR, PQ/PR is equal to QS/SR. (As shown below)



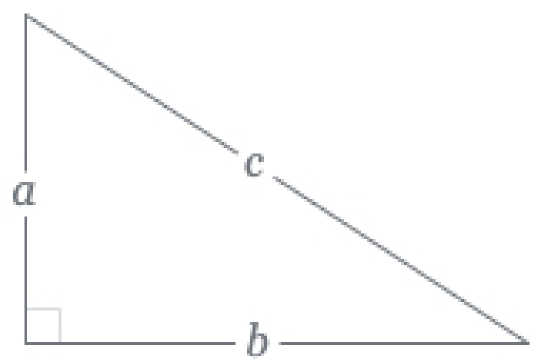
3 Pythagorean Theorem

One takeaway from the sum of angles is that in any given triangle, there may only be one angle greater than or equal to 90 degrees, although some triangles may have no such angles.

Triangles with one 90 degree angle are called right triangles.

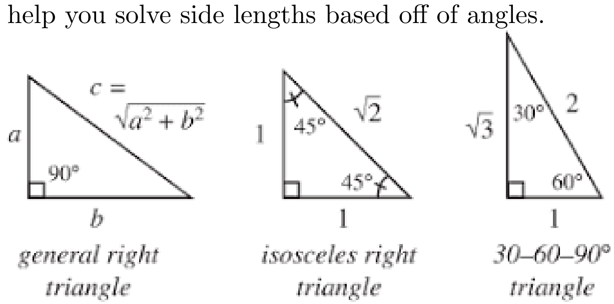
The most famous theorem related to right triangles is the Pythagorean Theorem, which states that a triangle is a right triangle if and only if its sides a, b, c, a $\leq$ b $\leq$ c, satisfy $a^2 + b^2 = c^2$. Since c is the largest side, it must be the hypotenuse and therefore, across from the 90 degree angle.

Remember, the Pythagorean Theorem only works with **right triangles**!



Cases of integer side lengths on a right triangle are called Pythagorean triples, such as (3, 4, 5) and (5, 12, 13).

There are a few special right triangles, which have standard formulas which help you solve side lengths based off of angles.



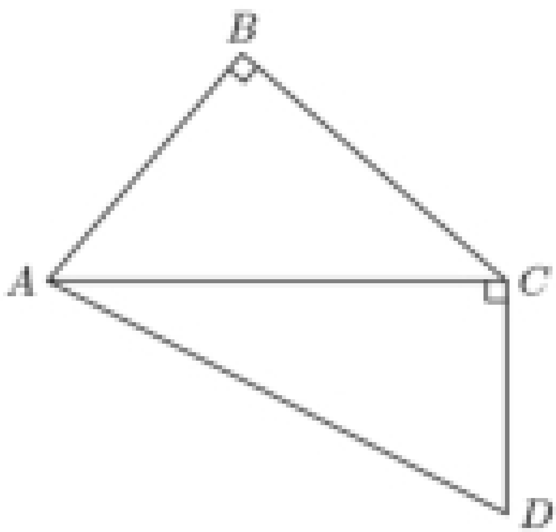
From the Pythagorean Theorem, we can also say that:

$a^2 + b^2 > c^2$ when the triangle is acute
$a^2 + b^2 < c^2$ when the triangle is obtuse

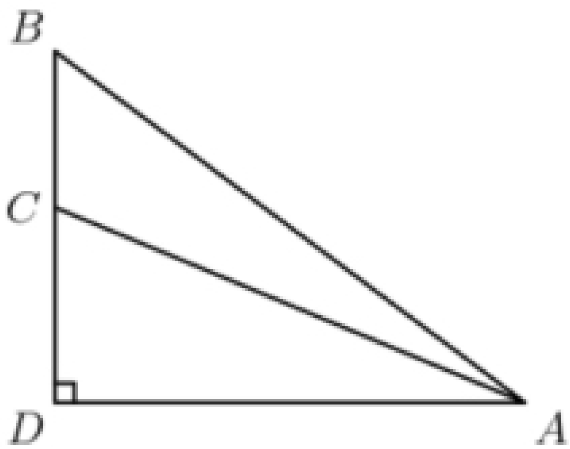
4 Practice

Two sides of a right triangle have the lengths 4 and 5. What is the product of the possible lengths of the third side?

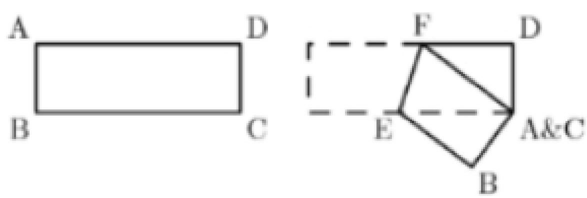
In quadrilateral $ABCD$, angle B is a right angle, diagonal AC is perpendicular to CD, $AB = 18$, $BC = 21$, and $CD = 14$. Find the perimeter of $ABCD$.



What is the area of ABC if $AC = 13$, $AB = 15$, and $DC = 5$?

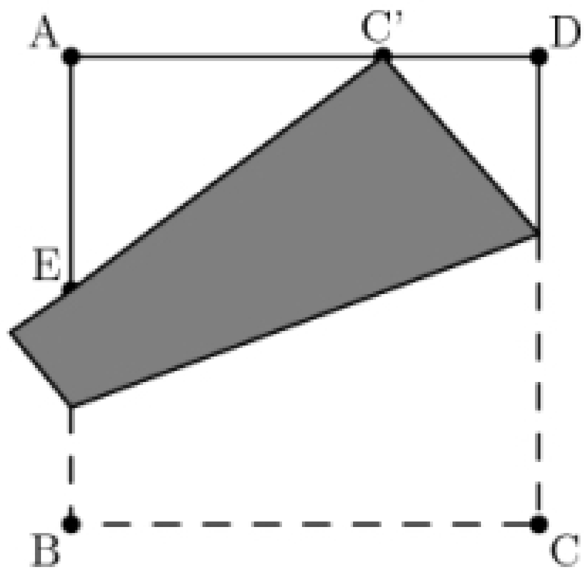


In the rectangle ABCD, AB = 3 and BC = 9. The rectangle is folded so that points A and C coincide, forming the pentagon ABEFD. What is the length of segment EF? Express your answer in the simplest radical form.

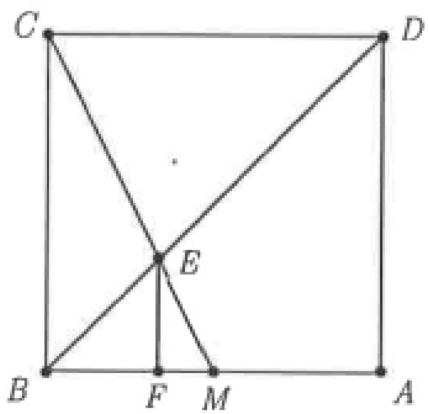


What is the volume of a tetrahedron ABCD with edge lengths $AB = 2$, $AC = 3$, $AD = 4$, $BC = \sqrt{13}$, $BD = 2\sqrt{5}$, and $CD = 5$?

A square piece of paper has side length 1 and vertices A, B, C, and D in that order. As shown in the figure, the paper is folded so that vertex C meets edge AD at point C, and edge BC intersects edge AB at point E. Suppose that $CD = 1/3$. What is the perimeter of triangle AEC?



Suppose that $ABCD$ is a square, with midpoint M of AB and intersection E of CM and BD. What is EB/EM?



Each of the sides of a square S_1 with area 16 is bisected, and a smaller square S_2 is constructed using the bisection points as vertices. The same process is carried out on S_2 to construct an even smaller square S_3. What is the area of S_3?

ABC is an equilateral triangle, and $ADEF$ is a square. If D lies on side AB and E lies on side BC, what is the ratio of the area of the equilateral triangle to the area of the square?

The radius of the inscribed circle of a triangle is equal to 1 and the lengths of its sides are integers. Prove that these integers are equal to 3, 4, 5.

The lengths of all the sides of a right triangle are integers and the greatest common divisor of these integers is equal to 1. Prove that the legs of the triangle are equal to $2mn$ and $m^2 - n^2$ and the hypotenuse is equal to $m^2 + n^2$, where m and n are integers.

Triangles ABC and BCD are such that either their corresponding angles are equal or their sum is equal to 180. Prove that the corresponding angles are equal, actually.

Prove that the area of a triangle is equal to $\sqrt{p(p-a)(p-b)(p-c)}$, where p is semiperime- ter (Heron's formula.)

a) Indicate two right triangles from which one can compose a triangle so that the lengths of the sides and the area of the composed triangle would be integers.
b) Prove that if the area of a triangle is an integer and the lengths of the sides are consecutive integers then this triangle can be composed of two right triangles the lengths of whose sides are integers