ORMC Beginners 2 **Winter 2024**

Vectors and Physics 4

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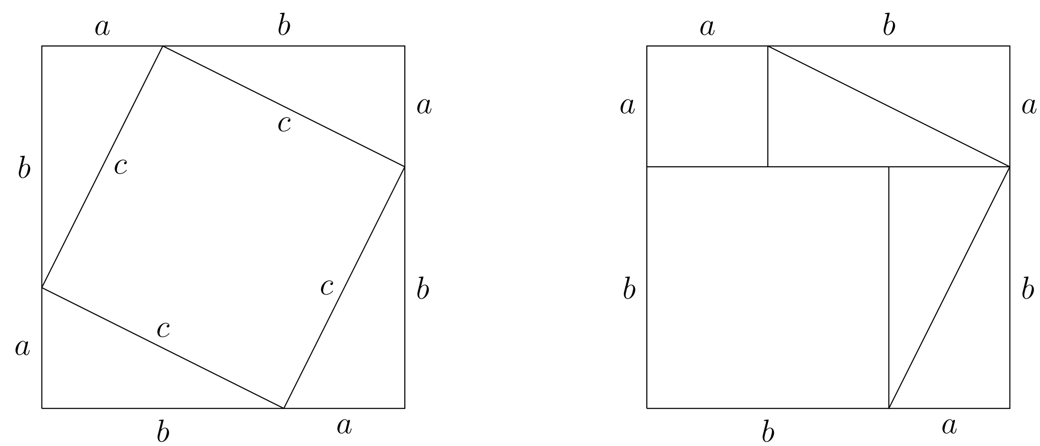
Warm-up

Problem 1 *We used the following two tilings of the square with a side length $a+b$ to prove the Pythagoras' Theorem.*



However, we have left a hole in our proof. Namely, we have not proved that the quad with a side length c on the left-hand-side picture is a square. Please prove it now.

Problem 2 *How many numbers from 0 to 99 contain the digit 3?*

Problem 3 *How many numbers from 0 to 999 contain the digit 3?*

Problem 4 *Can a triangle in the Euclidean plane have a side that is twice as long as one of the remaining two sides and twice as short as the other? Why or why not?*

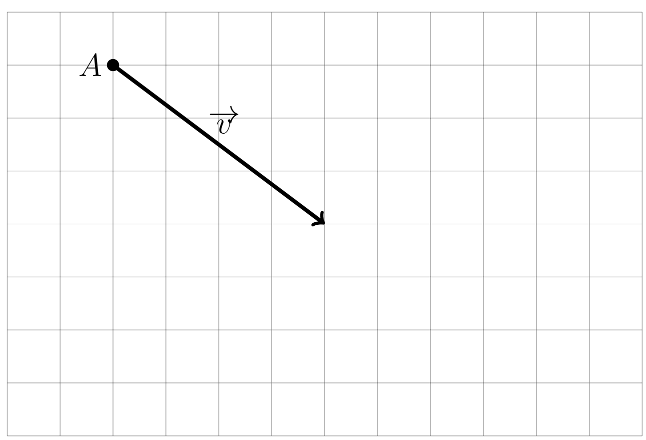
Vectors and Newtonian laws of motion

The First Law

The net force (the vector sum of all the forces acting on an object) is zero if and only if the velocity vector of the object does not change with time.

In other words, if no force acts on a body, then the body either remains at rest or moves along a straight line at a constant speed. The other way around, if the body does not move or moves along a straight line at a constant speed, then the vector sum of all the forces acting on the body is zero.

Problem 5 *There are no forces acting on the object A below. The current velocity of the body, in metres per second, is represented by the vector $\overrightarrow{v}$. Draw the position of the object two seconds later.*



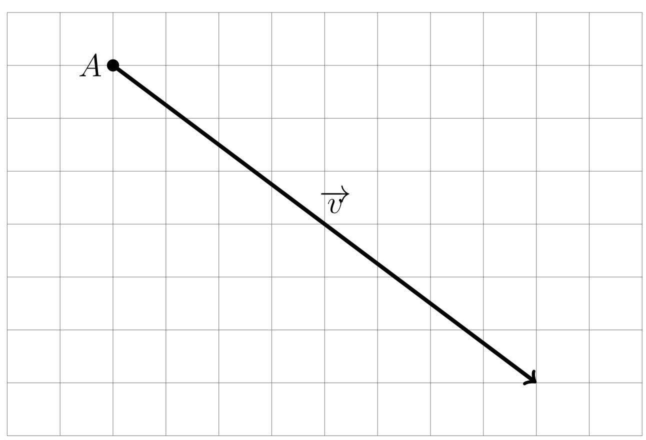
The sides of the grid squares on the picture above are one metre long. Find the speed of the motion. (Please continue to the next page.)

$$|\overrightarrow{v}| =$$

What distance would the object cover in two seconds?

$$d =$$

Problem 6 *There are no forces acting on the object A below. The current velocity of the body, in miles per hour, is represented by the vector $\overrightarrow{v}$. Draw the position of the object half an hour later.*



The sides of the grid squares on the picture above are ten miles long. Find the speed of the motion. (Please continue to the next page.)

$$|\overrightarrow{v}| =$$

What distance would the object cover in half an hour?

$$d(.5) =$$

What distance would the object cover in three hours?

$$d(3) =$$

Captain Solo's[1] ship, *Millennium Falcon*, chased by Imperial fighters, takes a desperate jump through space, turning up at the fringe of the universe. There are no stars, black holes, planets, asteroids, or other objects in sight. The *Falcon* is surrounded by nothing but dark void, so no gravitational force acts on the vessel.

Millennium Falcon

[1]One of the main characters of the popular *Star Wars* movies.

Tired from the battle, Captain Solo shuts down the engines, turns on his favourite music, grabs a can of the *Galactic Stout* ale from the ship's fridge, and takes a three hour rest. In the meantime, the *Falcon* moves from point A to point B on the picture below.

Problem 7 *Time is measured in hours. Draw the spaceship's velocity vector on the grid above.*

Problem 8 *The side length of the above grid squares is* $10,000\sqrt{5}$ *miles. What is the speed of the vessel in mph (miles per hour)? What is the distance between A and B?*

$$|\overrightarrow{v}| =$$

$$d(A, B) =$$

Acceleration

Acceleration is a vector that describes how velocity changes with time. For a constant acceleration,

$$\overrightarrow{v_t} = \overrightarrow{v_0} + t\,\overrightarrow{a}. \tag{1}$$

The velocity vector $\overrightarrow{v_t}$ at time t is the vector sum of the original velocity vector $\overrightarrow{v_0}$ and the acceleration vector $\overrightarrow{a}$ multiplied by the time t.

The unit of acceleration is one metre per second squared. The corresponding change of speed in one second is

$$1\frac{m}{s^2} \times 1s = 1\frac{m}{s}.$$

When the motion is one-dimensional, acceleration can be treated as a number, not a vector. After all, (real) numbers are nothing but one-dimensional vectors!

For example, it takes a popular Toyota Sienna minivan driving on a dry straight road in good condition nearly three seconds to decelerate from 60 mph ($\approx 26.8\frac{m}{s}$) to a complete stop. In this case, the acceleration equals

$$\frac{(0 - 26.8)\frac{m}{s}}{3s} \approx -8.9\frac{m}{s^2}.$$

In other words, the speed is reduced by about $8.9\frac{m}{s}$ every second.

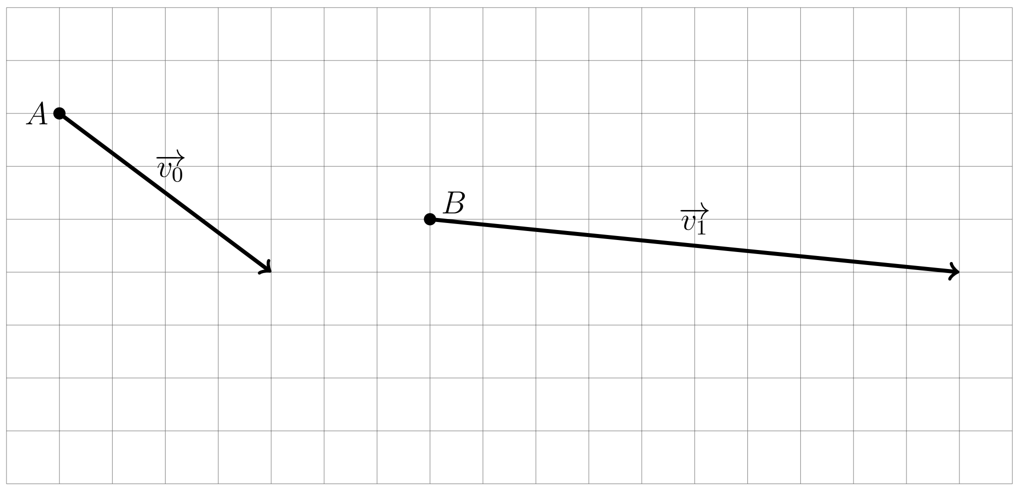
Problem 9 *It takes a Toyota Sienna minivan driving on a dry straight road in good condition seven seconds to accelerate from zero to 60 mph. Find the acceleration in metres per second squared.*

$$a = \qquad\qquad \frac{m}{s^2}$$

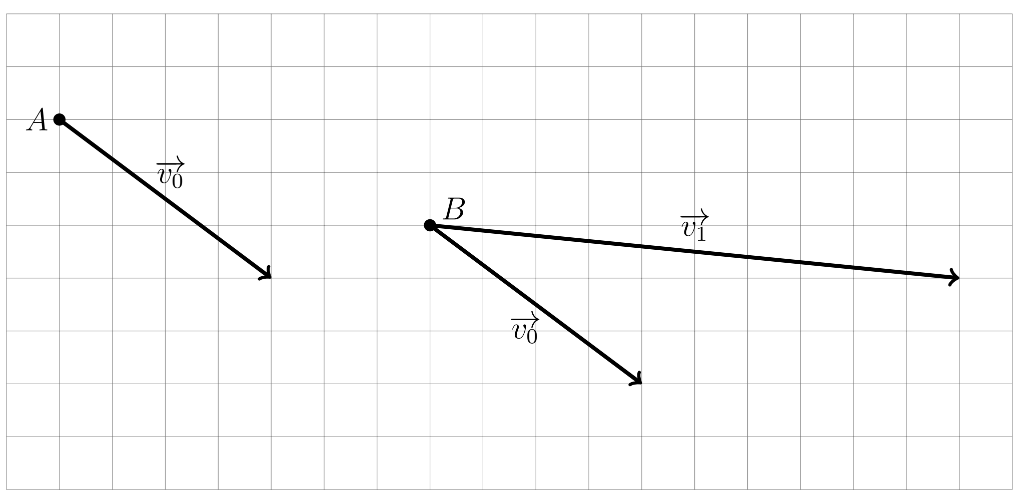
Problem 10 *The gravitational pull of our planet accelerates all the bodies near its surface at the rate of* $9.8\frac{m}{s^2}$*. A ball is thrown vertically from the ground level with the speed of* $29.4\frac{m}{s}$*. How many seconds later will the ball hit the ground?*

Note once again that in Problems 9 and 10, as well as in the example preceding them, the motion is one-dimensional. In this case, both velocity and acceleration are one-dimensional vectors, a.k.a. (real) numbers. In general, velocity and acceleration are vectors, not numbers! (Please see the example below.)

Example 1 *Moving with a constant acceleration, a body changes its position from point A to point B in one second. The velocities of the motion, in metres per second, are represented by the vectors $\overrightarrow{v_0}$ and $\overrightarrow{v_1}$. Find the acceleration vector.*

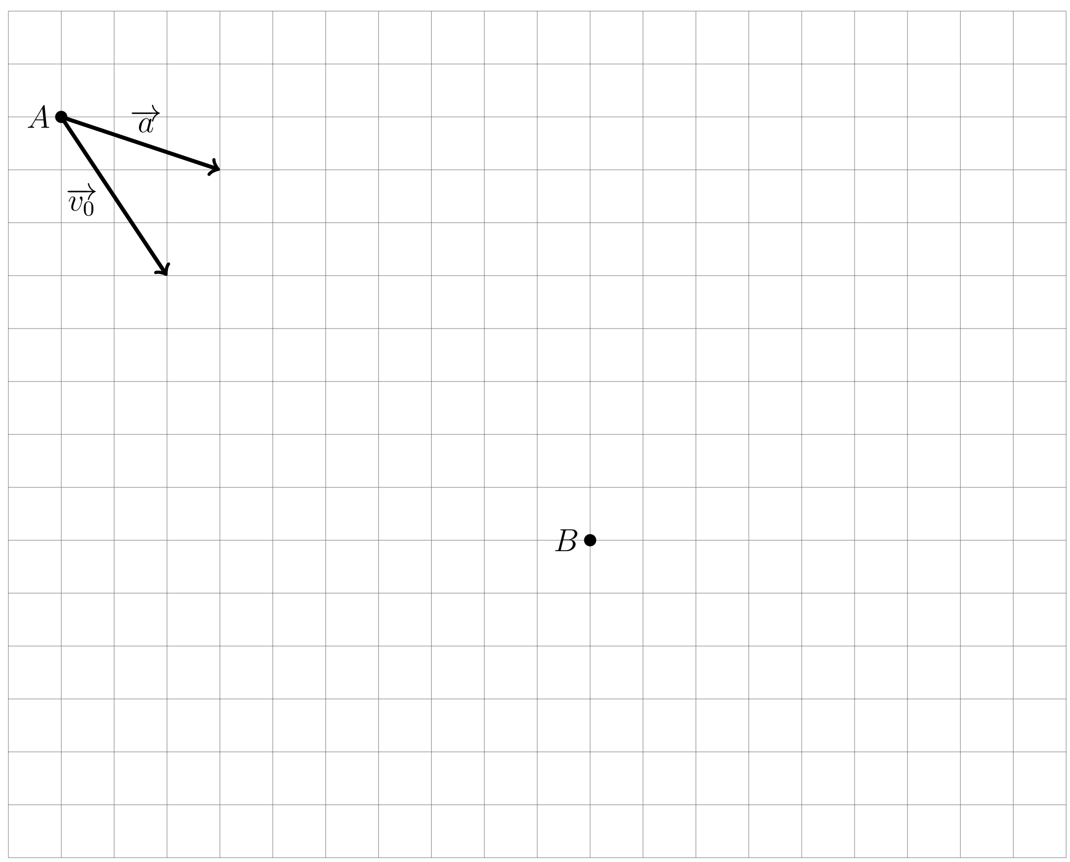


According to formula 1, $\overrightarrow{v_1} = \overrightarrow{v_0} + 1\ sec\ \times \overrightarrow{a}$.



Problem 11 *To complete Example 1, please draw the acceleration vector on the picture at the bottom of the previous page.*

Problem 12 *At the initial moment of time, the body at point* A *is moving with the velocity vector* $\overrightarrow{v_0}$*, measured in metres per second. The constant acceleration acting on the body is represented by the vector* $\overrightarrow{a}$*, measured in metres per second squared. Two seconds later, the body is located at point* B*. Draw its velocity vector at the moment.*



The problem continues on the next page.

The side length of the grid squares on the picture above is one metre. Find the speed of the body at point B.

$$|\overrightarrow{v_2}| =$$

The Second Law

$$\overrightarrow{F} = m\overrightarrow{a} \tag{2}$$

Here $\overrightarrow{F}$ is the net force acting on a body of a constant mass m and $\overrightarrow{a}$ is the acceleration the body undergoes as a result of the action. The mass measures the *inertia* of the body – the heavier the body the harder it is to change the way it moves.

In modern day physics, the force is measured in *newtons*. One newton is the force needed to accelerate 1 kilogram of mass at the rate of 1 metre per second squared.

$$1N = 1kg\,\frac{m}{s^2} \tag{3}$$

In the US and Britain, they also use an outdated unit, one pound-force, 1 lbf. The pound-force is defined as the gravitational force exerted on a mass of one pound on the surface of the Earth. Our home planet is not a perfect sphere. What's more, its density varies for different locations on the globe. The gravitational pull of the Earth is not uniform. Hence, 1 lbf is only practical to use in calculations that do not require high precision.

The following formula helps to convert pounds of force to newtons.

$$1\ lbf \approx 4.448\ N \tag{4}$$

Problem 13 *Captain Solo has rested. It's time to save the galaxy, again! He turns on the engines. The engines of the Millennium Falcon generate 200,000 kN (kilonewtons, 1 kN = 1,000 N) of force. The mass of the ship is 400 metric tons (1 metric ton = 1,000 kg). The mass of the fuel spent during the short period of time in consideration is negligible compared to the mass of the ship, so we consider the latter as a constant.*

What is the acceleration of the vessel in metres per second squared?

$a =$

How long would it take the Falcon to reach the speed of $10,000\frac{m}{s}$*?*

$$t =$$

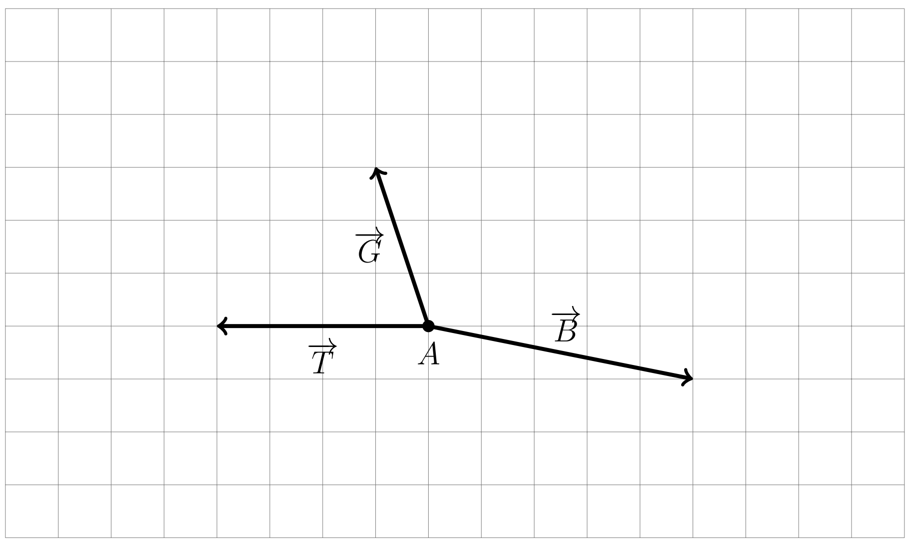
What is the acceleration of the vessel in g's $(g = 9.8\frac{m}{s^2})$*?*

$$a =$$

On Earth, Captain Solo's weight would have been 80 kg (176.6 lbs). What is his weight during the acceleration?

$$w =$$

Problem 14 *The Falcon, at point A at the moment, is trying to escape from the Death Star that in its turn is trying to arrest the ship using the attracting beam. The thrust of the Falcon's engines, 200,000 kN in total, is represented by the vector* $\overrightarrow{T}$. *The force of the beam is represented by the vector* $\overrightarrow{B}$. *In addition, the neighbouring star exerts the gravitational pull* $\overrightarrow{G}$ *on the ship. Draw the vector of the net force acting on the vessel.*



Recall that the mass of the ship is 400 metric tons. What is the acceleration, in metres per second squared, the ship undergoes at the moment?

$a =$

The purpose of the daring mission was to find out the maximal force of the attracting beam. However, Captain Solo is not too good with vectors. Please help him complete the mission.

$$|\overrightarrow{B}| =$$

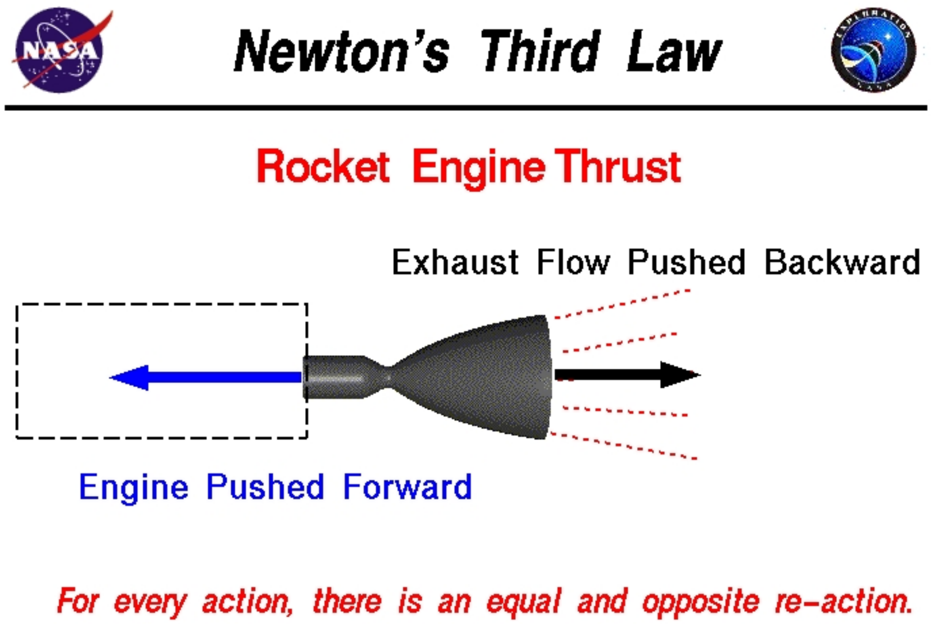
The Third Law

When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction to the force exerted on it by the first body.

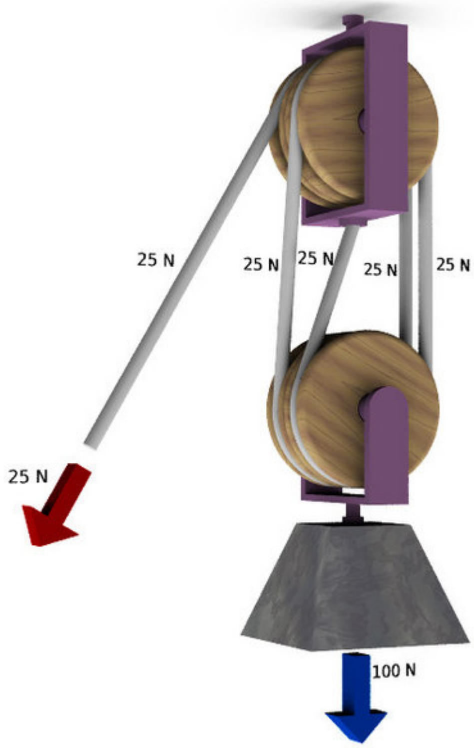
The vector form makes the Third Law very clear.

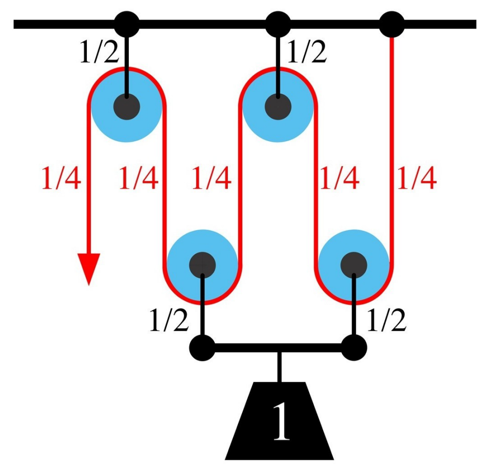
$$\overrightarrow{F}_{reaction} = -\overrightarrow{F}_{action} \tag{5}$$

You will find a few examples below. The first is from the website of NASA, National Aeronautics and Space Administration.

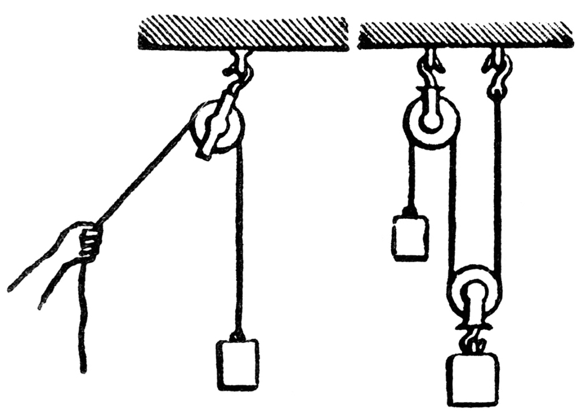


The other example shows the workings of one of the simplest, and most important, machines invented by the humanity, a pulley. Below you will find a picture and schematics of a *double tackle*, the device with the *advantage* four.





Problem 15 *What is the advantage of the pulley on the left-hand side of the below picture? On the right-hand side?*



Problem 16 *Draw the scheme of a pulley with the advantage three.*

If you are finished doing all the above and there remains some time...

Problem 17 *Find the area of an equilateral triangle with the side length a.*

Problem 18 *Does there exist an equilateral triangle in the Euclidean plane such that all of its vertices have integral coordinates? Why or why not?*