The Math Behind Bookmaking: Notes and Solutions

1. \$30
2. Give those who have a less likely probability a higher payout.
3. $P(A) = \frac{y}{x+y}$
4. x/y odds $= \frac{y}{x+y}$
5. The odds form a contest.
6. $4.5, 22\%, 1.25, 80\%, 3.67, 27.3\%$
7. $11, 2.2, 1.87$
8. Decimal odds, treat these as payout multipliers.
9. $7.32, 72.6$
10. Maintain probability ratios but increase implied probabilities to decrease payouts.
11. $\frac{1}{5}, \frac{2}{5}, \frac{6}{5}$
12. Profit is \$20.
13. $14.4\%, 22.4\%, 31.2\%$. Decimal odds are $(1.87)^n$, where n is the number of legs.
14. Convert multipliers to implied probabilities and use Problem 13. Straight bets are more profitable since the overround is less. A loss is what the books win, which is represented by overround.
15. Suppose x is the hit rate. Then set expected value to be greater than 0, and the expressions should be $x > \sqrt{\frac{1}{3}} \implies x \approx 57.7\%$ for 2-legs, $x > \sqrt[3]{\frac{1}{5}} \implies x \approx 58.5\%$ for 3-legs, and $x > \sqrt[4]{\frac{1}{10}} \implies x \approx 57.7\%$ for 4-legs.
16. 28
17. Approximately $\frac{1}{276000}$. $(1.67)^{28} \times 2^4$ is the multiplier, which can be converted to implied probability.
18. Solve using the strategy from Problem 17 and consider the binomial theorem.
19. Decimal odds should be between $\binom{28}{4}(2.5)^4(1.67)^{24}(2)^4$ and $\binom{28}{5}(2.5)^5(1.67)^{23}(2)^4$.
20. It could help guarantee that the bets would be profitable, rather than risking the third leg for a loss.
21. Shifted lines, high probability of original bet hitting, etc.
22. Putnam 2018 B6.
23. Putnam 2013 A1.
24. Putnam 2012 B3.