Exploring the City of Descartes I

November 9, 2012

Part I

Review: (x, y) coordinates on the plane

Please DO NOT use the geoboards in this part

1. For a point (x, y) on the plane, the first number is called x-coordinate and the second one is called the y-coordinate.
   For example, for point $P = (-1, 4)$ we have $x = -1$; $y = 4$.

   (a) Find the x-coordinate of the following points:

   i. $A = (3, 2) \Rightarrow x =$;
   ii. $B = (4, -5) \Rightarrow x =$;
   iii. $C = (-7, -9) \Rightarrow x =$;

   (b) Find the y-coordinate of the following points:

   i. $A = (3, 2) \Rightarrow y =$;
   ii. $B = (4, -5) \Rightarrow y =$;
   iii. $C = (-7, -9) \Rightarrow y =$;

Part II

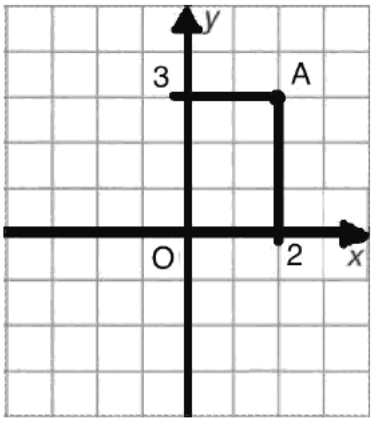
The City of Descartes

Please DO NOT use the geoboards in this part of the class

René Descartes was a French mathematician, philosopher and writer. Among his many accomplishments, he developed a very convenient way to describe positions of points on a plane. This method was very important for future development of mathematics and physics. We will start learning about this invention today.

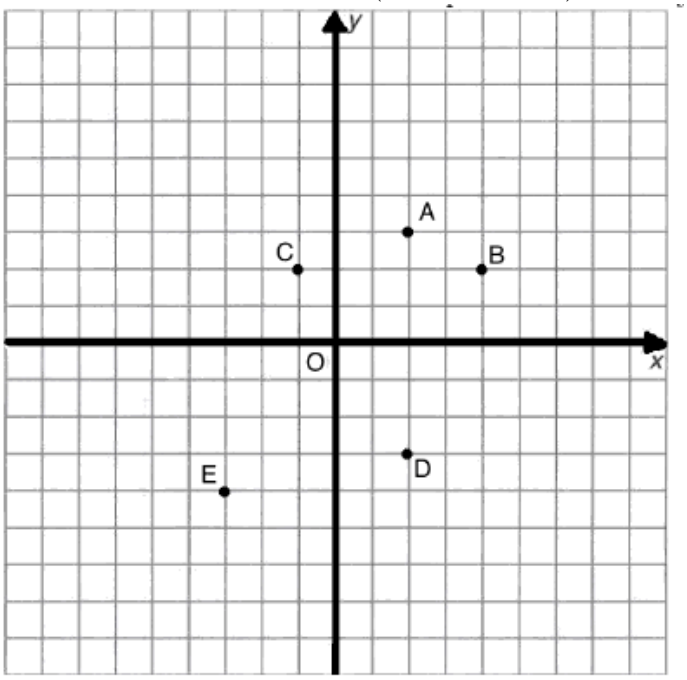
The city of Descartes is a plane that extends infinitely in all directions:

- The center of the city is marked by point O.
- The horizontal (West-East) line going through O is called the x-axis.
- The vertical (South-North) line going through O is called the y-axis.
- Each house in the city is represented by a point which is the intersection of a vertical and a horizontal line. Each house has an address which consists of two whole numbers written inside of parenthesis. For example, $(2, 3)$ is an address of the house as shown below:



- The first number tells you the x-value. However, it is also the distance to the y-axis. The distance is *positive* if you are on the right of the y-axis. The distance is *negative* if you are on the left side of the y-axis.
- The second number tells you the y-value. However, it also is the distance to the x-axis. The distance is *positive* if you are above the x-axis. The distance is *negative* if you are below the y-axis.

1. Let's find the addresses(coordinates) of several points in the city:



(a) Point O has address (,);

(b) Point A has address (,);

(c) Point B has address (,);

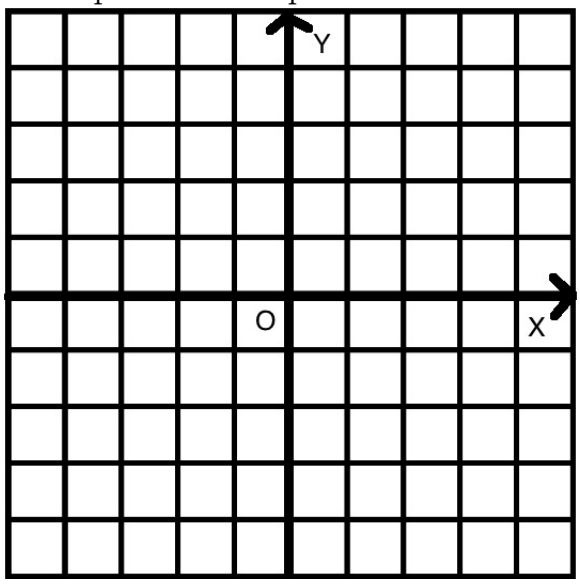
(d) Point C has address (,);

(e) Point D has address (,);

(f) Point E has address (,);

(g) The midpoint between A and D has address (,);
(Hint: The midpoint is the point on segment AD which is the same distance to A as it is to D. You can think of it as the "middle".)

2. Let's plot several points whose coordinates are given:

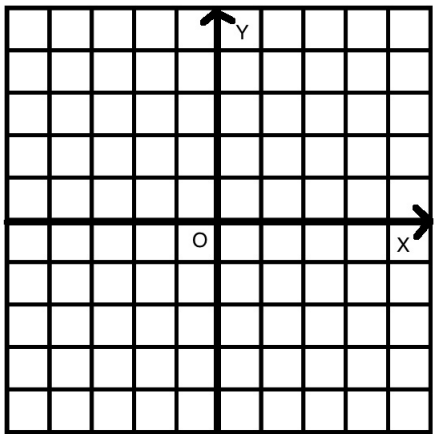


   (a) Plot point F with address $(1, 4)$;
   (b) Plot point G with address $(4, 1)$;
   (c) Plot point H with address $(5, 3)$;
   (d) Plot point J with address $(2, 5)$;
   (e) Plot point K with address $(0, 2)$;
   (f) Plot point L with address $(3, 0)$.

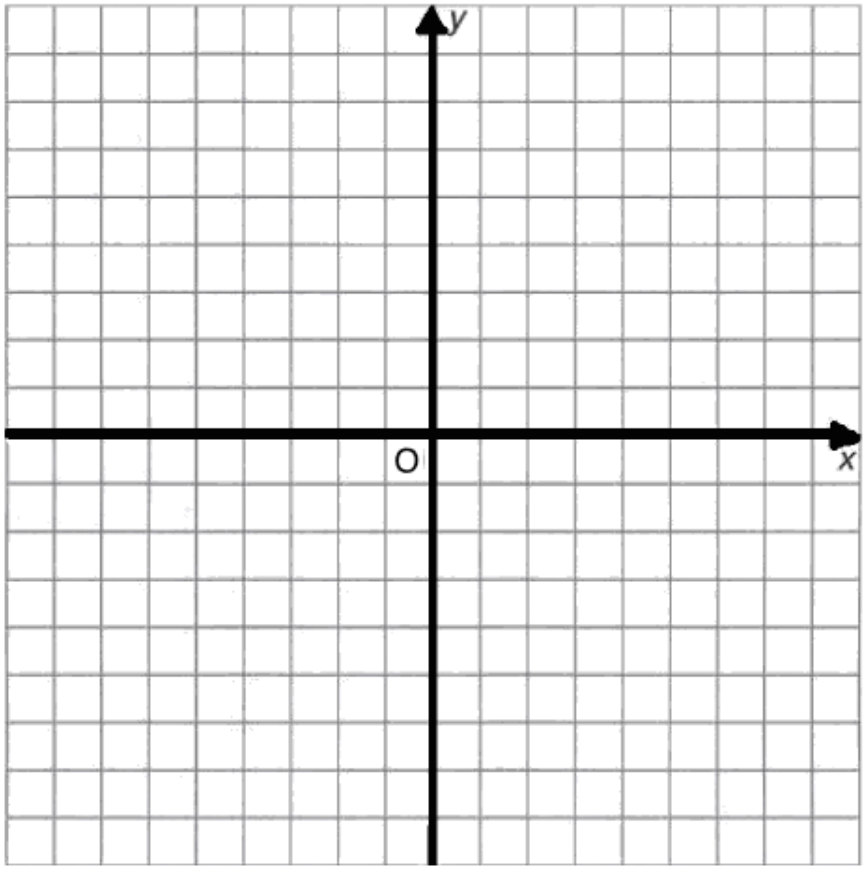
3. Let n be any whole number. Draw the following sets of points on the coordinate plane below. Use pencils of different colors to draw different sets:

   (a) all points with addresses $(n, 0)$:
   (b) all points with addresses $(0, n)$:
   (c) all points with addresses $(n, 5)$:
   (d) all points with addresses $(5, n)$:
   (e) with addresses (n, n)

(f)



4. Plot the points and find the distance between the following points:



(a) $(4, 3)$ and $(4, 7)$;

Distance=

(b) $(-1, 3)$ and $(-1, 5)$;

Distance=

(c) $(6, 5)$ and $(8, 5)$;

Distance=

(d) $(5, -2)$ and $(7, -2)$;

Distance=

Part III

Reflections, Directions, and Squares

Please use the Geoboards in this section

1. **Reflection in x-axis**

   On the geoboard, put in blue pegs to mark the following points:

   $$(2,3), \qquad (5,1), \qquad (-6,2), \qquad (-3,4).$$

   Imagine that the x-axis is a mirrow. Reflect each of the points in this mirror and put a red peg to indicate the reflection. Then record the coordinates of the reflected points:

   (a) $(2,3) \longrightarrow (\quad , \quad)$
   (b) $(5,1) \longrightarrow (\quad , \quad)$
   (c) $(-6,2) \longrightarrow (\quad , \quad)$
   (d) $(-3,4) \longrightarrow (\quad , \quad)$
   (e) What would be the reflection of the point with coordinates (x,y)?
   $(x,y) \longrightarrow (\quad , \quad)$
   (f) Explain what happens to a point if you reflect it twice in the x-axis.

   Remove all the red pegs from the geoboards

2. **Reflection in y-axis**

   On the geoboard, put in blue pegs to mark the following points:

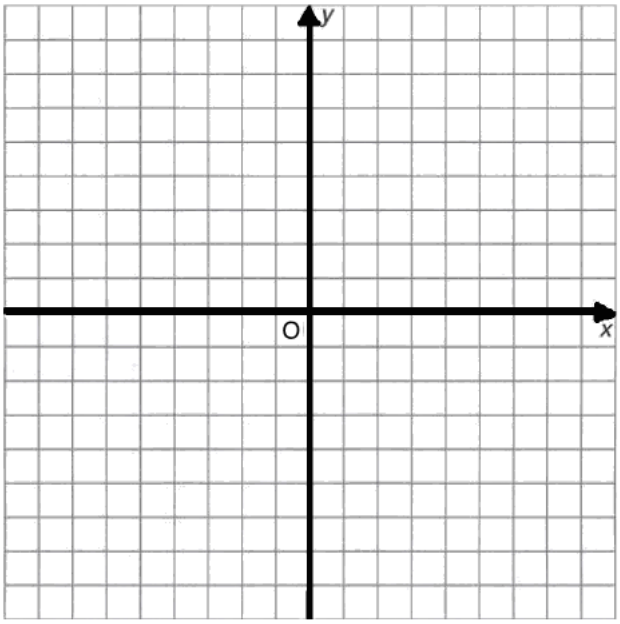
   $$(2,3), \qquad (5,1), \qquad (-6,2), \qquad (-3,4).$$

   Imagine that the y-axis is a mirrow. Reflect each of the points in this mirror and put a red peg to indicate the reflection. Then record the coordinates of the reflected points:

   (a) $(2,3) \longrightarrow (\quad , \quad)$
   (b) $(5,1) \longrightarrow (\quad , \quad)$
   (c) $(-6,2) \longrightarrow (\quad , \quad)$
   (d) $(-3,4) \longrightarrow (\quad , \quad)$
   (e) What would be the reflection of the point with coordinates (x,y)?
   $(x,y) \longrightarrow (\quad , \quad)$
   (f) Explain what happens to a point if you reflect it twice in the y-axis.

3. Remove all the pegs from the geoboard. Suppose you start at the point $(0, 0)$. Then

- Go north (up) for two units;
- Go east (right) for 5 units;
- Go south for 1 unit;
- Go west for 1 unit.



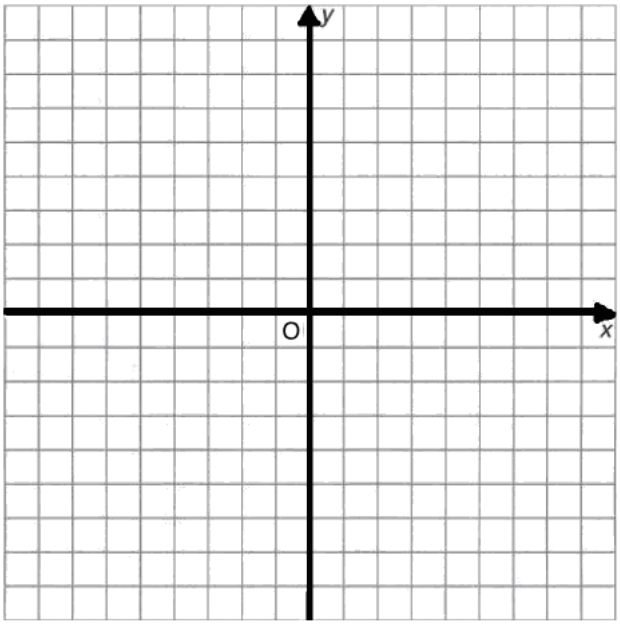
Where are you now? Draw the route and give the address of the final point.

4. Find a square such that

- two sides of the square are vertical and the other two are horizontal;
- Two of the vertices of a square are at points $(2, 3)$ and $(5, 3)$;

Hint: There is more than one solution, find all possible solutions.

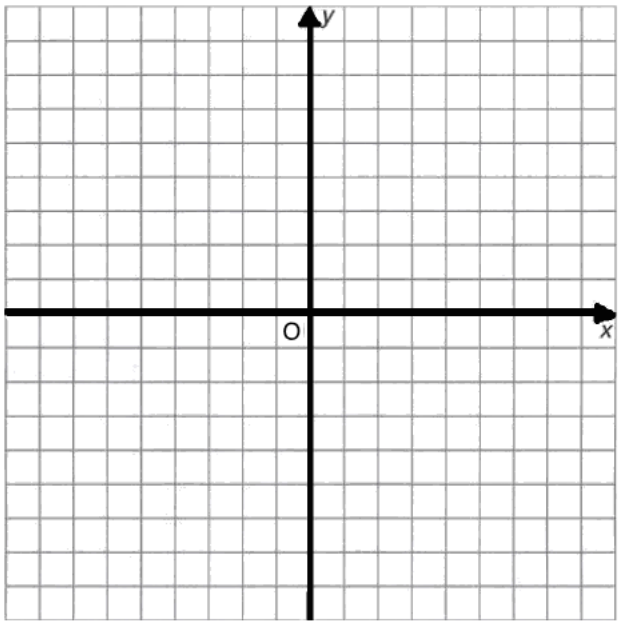
First, use the rubberbands and the geoboard to represent the square(s). Then, draw your solutions on the coordinate plane below.



For each solution, find the coordinates of the other two vertices of the square.

5. Two of the vertices of a square are at the points $(0, 0)$ and $(4, 4)$. Find the other vertices of this square. (First, use the rubberbands and pegs on the geoboard. Then draw your solution(s) on the coordinate plane below).

   Can you find a square with these vertices whose sides are *not* vertical and horizontal?



Part IV
Finishing Up

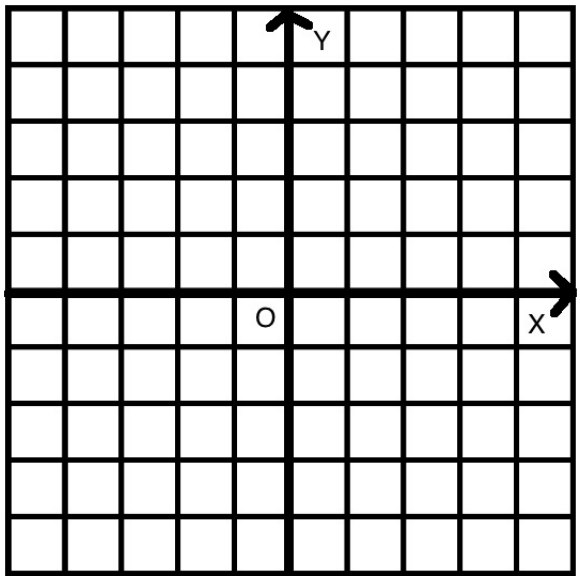
Please DO NOT use the geoboards in this section.

1. The houses of Amy, Ben, Cindy and Dan are vertices of a square:

- The center of this square is at the point $O = (0, 0)$;
- The length of each of the sides of this square equals to 4;
- Amy's house is directly to the north from Dan's house;
- Ben's house is east from Amy's house;

Denote the vertices of this square by A, B, C, D. Mark the houses (vertices) on the picture below and find their addresses:

$$\begin{aligned} A &= (\quad , \quad), \\ B &= (\quad , \quad), \\ C &= (\quad , \quad), \\ D &= (\quad , \quad). \end{aligned}$$



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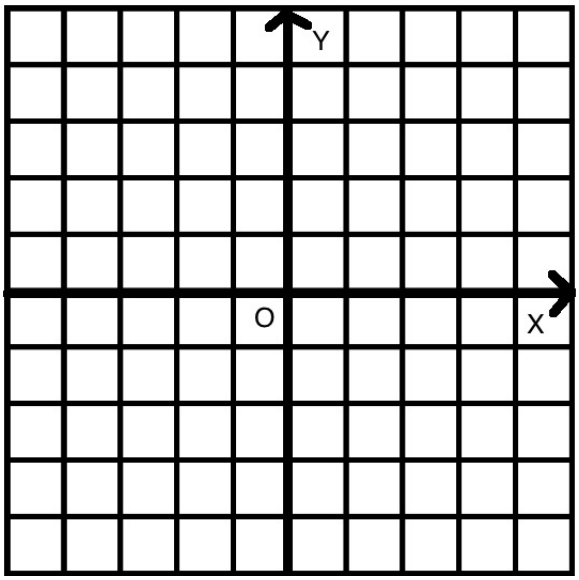
1. The houses of Eddie, Fred, George and Helen are also vertices a square:

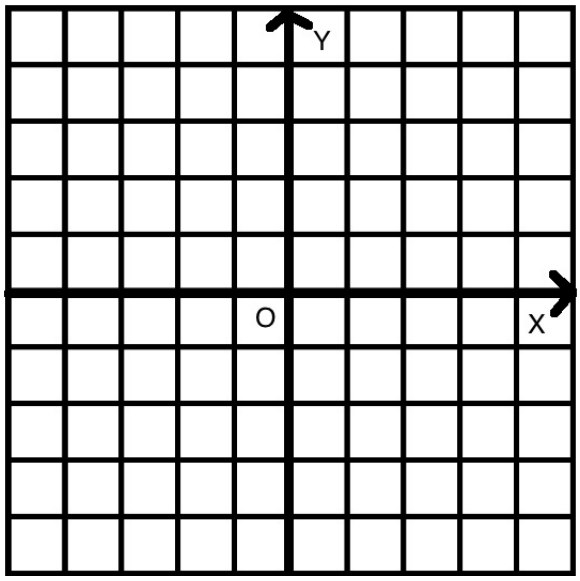
   - The center of this square is at the point $O = (0, 0)$;
   - The distance from O to any of these houses is 2;
   - George's house is east of Eddie's house;
   - Fred's house is north of Helen's house;
   - Denote the vertices of this square by E, F, G, H.

   Mark the vertices on the same picture and find their addresses:

$$\begin{aligned} E &= (\quad , \quad), \\ F &= (\quad , \quad), \\ G &= (\quad , \quad), \\ H &= (\quad , \quad), . \end{aligned}$$



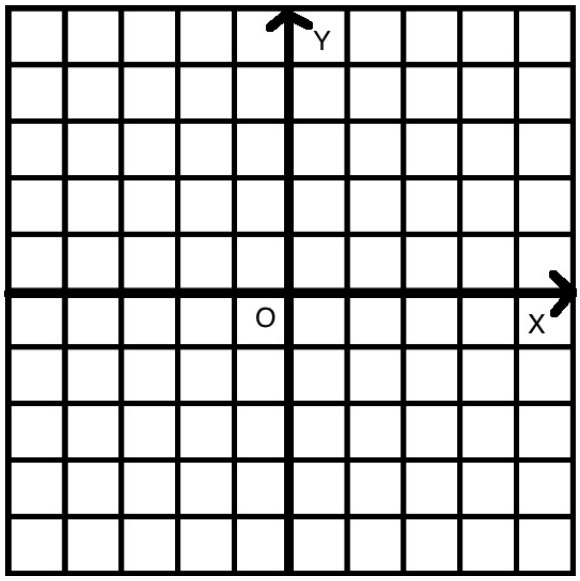
2. Plot several points and reflect them across the x-axis. (Imagine that x-axis is a mirror)



(a) Does the x-coordinate change when you reflect a point across the x-axis? If so, how?

(b) Does the y-coordinate change when you reflect a point across the x-axis? If so, how?

3. Plot several points and reflect them across the y-axis. (Imagine that y-axis is a mirror)



(a) Does the x-coordinate change when you reflect it across the y-axis? If so, how?

(b) Does the y-coordinate change when you reflect it across the y-axis? If so, how?