LAMC Junior Circle **November 11, 2012**

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Math salad

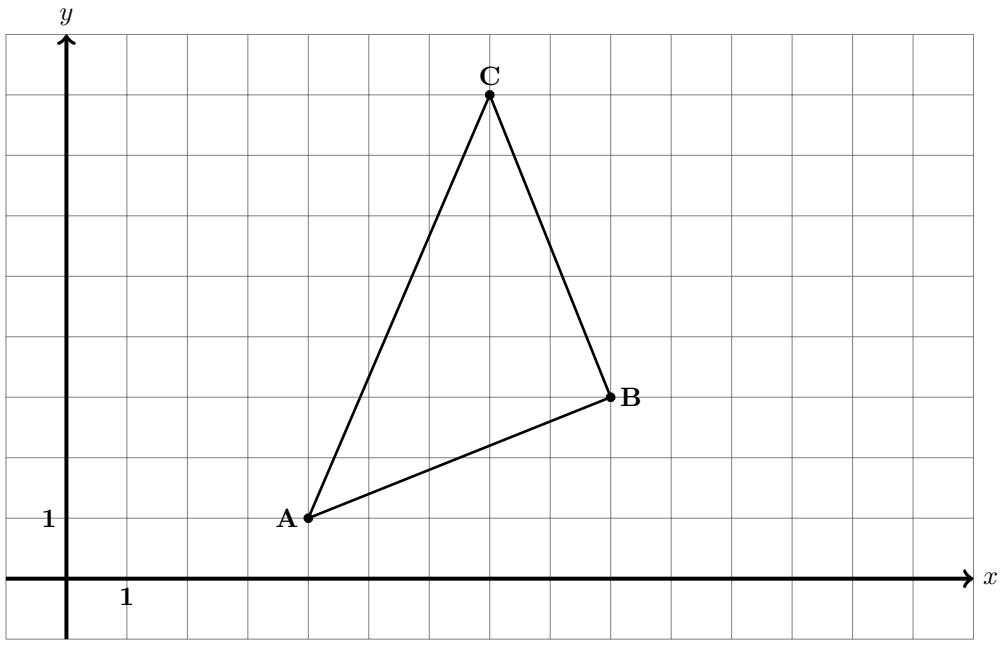
Problem 1

$$\sqrt{81} =$$

$$\sqrt[4]{81} =$$

$$\sqrt[3]{3} \equiv \qquad\qquad mod\ 5$$

Problem 2 *The final goal of the problem is to figure out whether the angle ABC is right. We will do it in the following steps.*



• *Find the area of the square built on the side AB of the above triangle.*

• *Find the area of the square built on the side BC of the above triangle.*

• *Find the area of the square built on the side AC of the above triangle.*

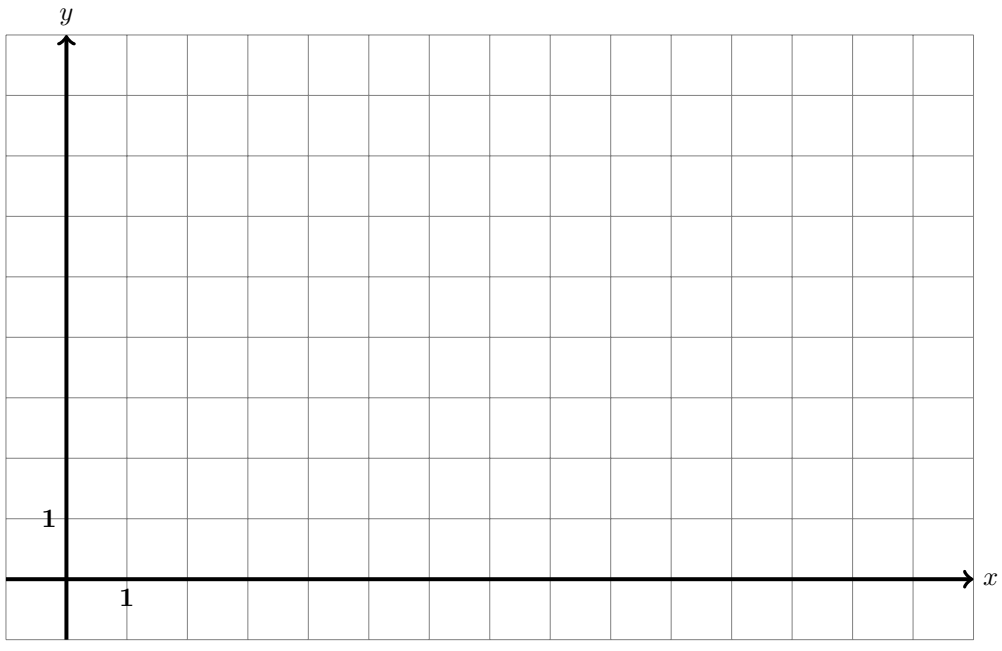
Is the angle ABC right? Why or why not?

Problem 3 *Imagine that you are an ancient Greek engineer. You need to help your fellow citizens construct a rectangular (90°) corner of the wall around your city. All you have for taking measurements is a rope and lots of helpers. How would you do it?*

Problem 4 *Draw the graph of the function*

$$y = 4$$

on the grid below.



Problem 5 *A digital clock shows time in hours and minutes and denotes whether the time is AM or PM. How much time daily does the displayed time have at least one 2?*

Problem 6 *Alice and Bob come to visit Charlie, who lives in an apartment building. The visitors do not remember the number of their buddy's apartment. Alice recalls that the number greater than the apartment number by 10 is a cube of some integer. Bob remembers that the number less than the apartment number by 10 is a square of another integer. What is the apartment number?*

Additional info: since they don't build huge apartment complexes in this country, you can safely assume that the apartment number is less than 1000.

Problem 7 *King Louis XIII*[1] *of France has set up a fencing competition between the best of the French Army swordsmen. Athos, Porthos, Aramis and D'Artagnan*[2] *have taken the first four places.*

D'Artagnan and the three musketeers.[3]

The sum of the places taken by Athos, Porthos and D'Artagnan equals 6. The sum of the places Athos and Aramis have taken equals 6, too. The place Athos has taken is superior to that of Porthos. What place did each of the musketeers take?

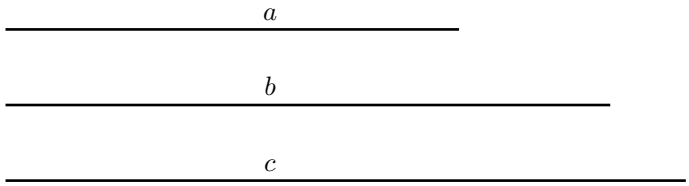
[1] 1601 – 1641

[2] The main characters of *The Three Musketeers* novel by Alexandre Dumas.

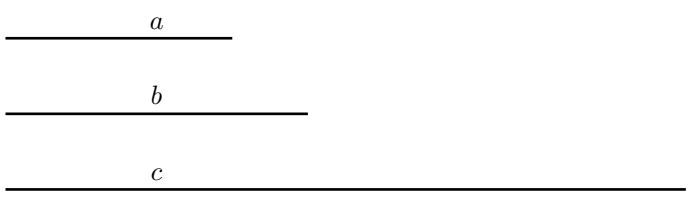
[3] The drawing by Maurice Leloir.

The triangle inequality and the shortest paths between points.

Problem 8 *Use a compass and a ruler to construct a triangle having the following sides in the space below.*



Problem 9 *Use a compass and a ruler to construct a triangle having the following sides in the space below.*



Can you? What's wrong?

The following theorem is known as the *triangle inequality*.

Theorem 1 *The length of a side of any triangle in the Euclidean plane is less than the sum of the lengths of the other two sides.*

Problem 10 *Prove the theorem.*

Definition 1 *A line connecting two points on a surface is called a geodesic line if it is a shortest line (there might be more than one) among all the lines connecting the points.*

Problem 11 *Use the triangle inequality to prove that straight lines are the geodesic lines in the Euclidean plane.*

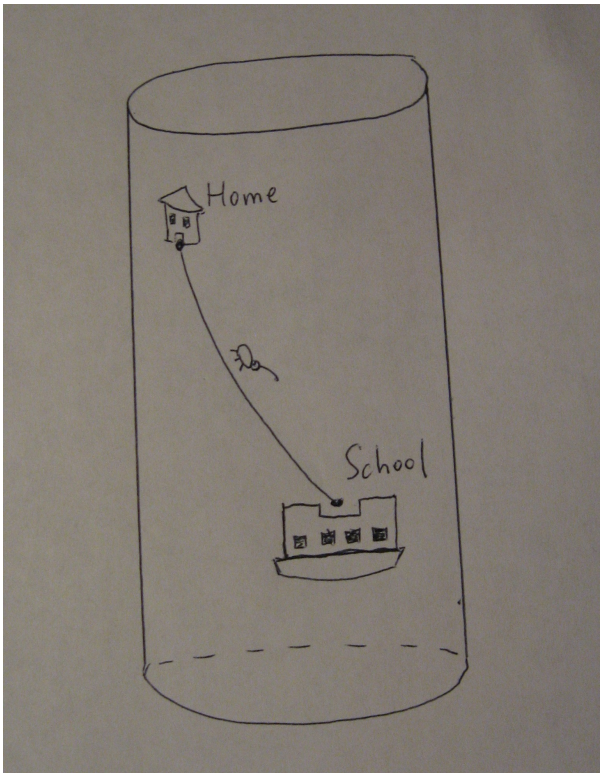
Euclid's Postulate 1 (modern version) *For any two different points in a Euclidean plane, there exists the unique straight line passing through them.*

Combining the first postulate with Problem 11 proves the following.

Theorem 2 *The shortest path between any two different points in a Euclidean plane is the (unique) segment of the straight line connecting them.*

In the final part of this lesson, we will figure out what the geodesic lines on a cylinder are and whether Postulate 1 holds for this surface.

Please take the sheet following this one, cut out the rectangle and glue it into a cylinder, so that the points H (Home) and S (School) appear on its outside surface. Imagine that the cylinder is inhabited by some tiny bugs, intelligent just like us. The bugs' children need your assistance. Please help them find the shortest way from Home to School.



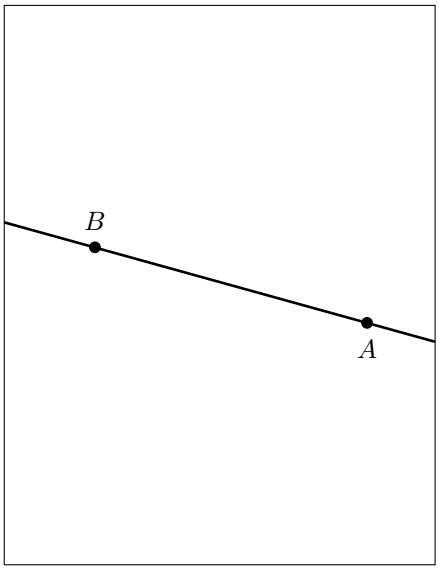
Problem 12 *Draw a geodesic line on the cylinder connecting the points H and S.*


S
H

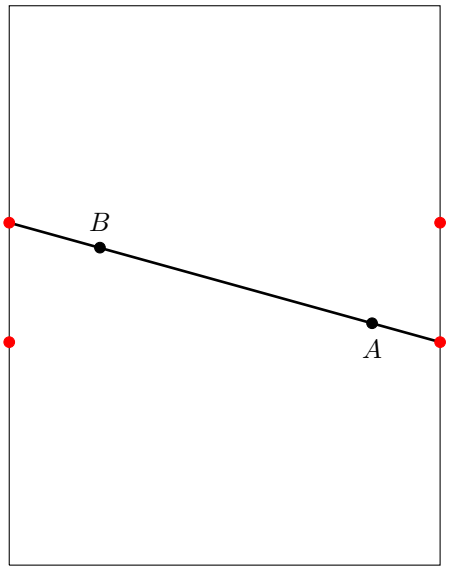
Problem 13 *Cut out the rectangle drawn on the opposite side of this sheet and glue it into a cylinder so that the points H and S appear on its outside surface. Draw a geodesic line connecting the points H and S on the cylinder. Is there only one? Why or why not? If you think that there are a few geodesic lines connecting the points, which one is the shortest?*


S
H

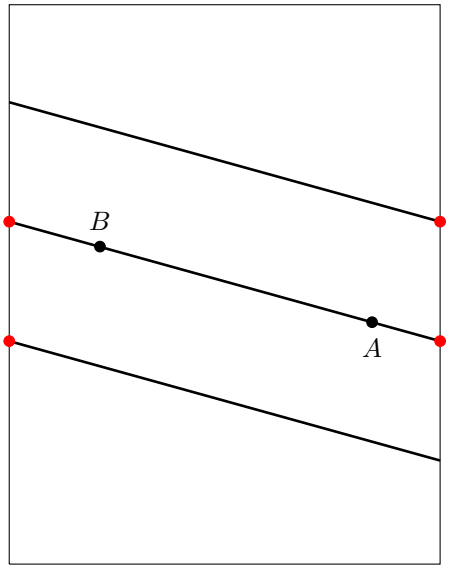
Let us investigate the behaviour of the geodesic line connecting the points A and B on the cylinder below.



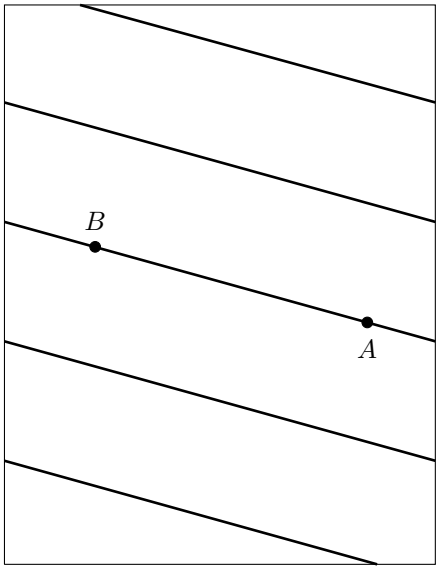
As we glue the sides together to form a cylinder, the left end of the segment will appear on the right hand side of the sheet while the right end will appear on the left.

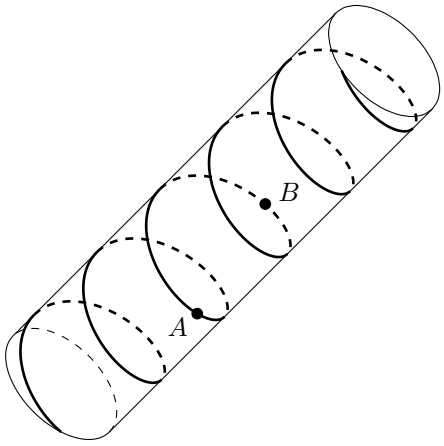


The line will continue going past the cut until it reaches the sides of the sheet again.



There, it will jump to the opposite sides of the cut one more time. And one more... We end up with the following picture.



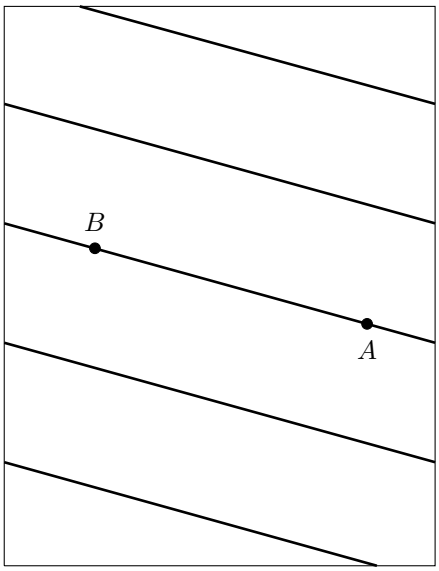


A geodesic line on a cylinder connecting two points in general position is called a *helix*.

Problem 14 *What kinds of helical objects can you see in everyday life?*

The shortest paths between points, helical lines are the "straight lines" on a cylinder.

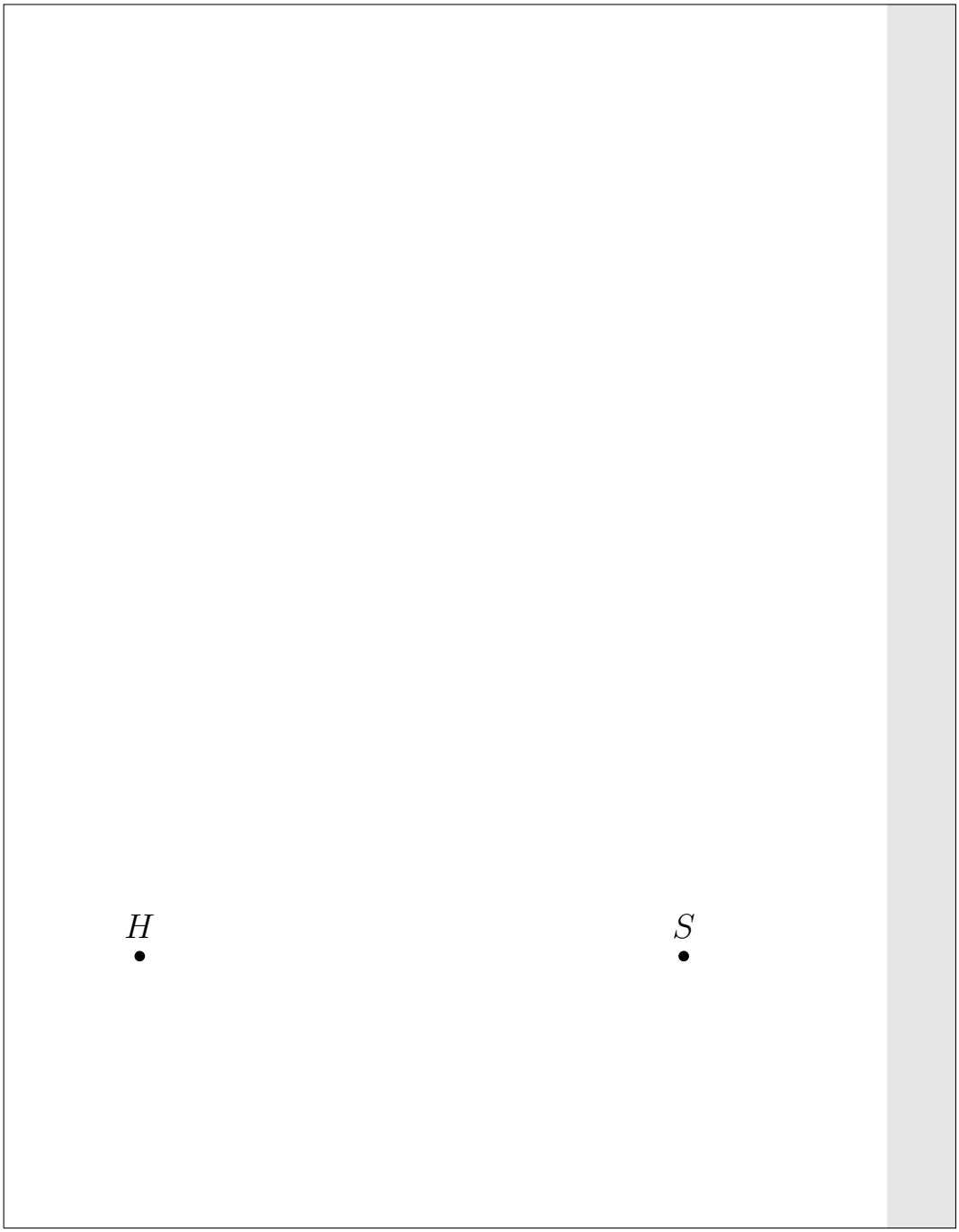
Problem 15 *Use the picture below to answer the following question. Does the cylinder satisfy Euclid's Postulate 1 (see page 9)? Why or why not?*



There exist two more types of geodesic lines on a cylinder. We will investigate them in the subsequent two problems.

Problem 16 *Cut out the rectangle on page 19 and glue it in a cylinder so that the points H and S appear on its outside surface. What is the geodesic line connecting the points?*

Problem 17 *Cut out the rectangle on page 21 and glue it in a cylinder so that the points H and S appear on its outside surface. What is the geodesic line connecting the points?*


H
S

- S

- H

Problem 18 *What types of geodesic lines are there on a cylinder?*

Problem 19 *Use a compass and a ruler to construct a pair of points on the cylinder below such that there is more than one shortest path connecting the points.*