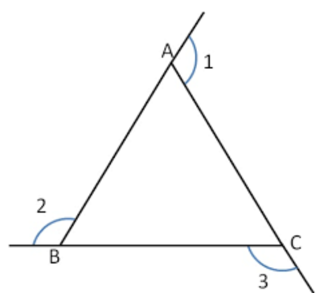
Intermediate 1 **March 10, 2024**

Competition!

1 10 Points Problems

Problem 1.1. *Name 3 congruence test for triangles in Euclidean geometry.*

Problem 1.2. *In the diagram below, suppose* $\angle 1 = \angle 2 = 125°$. *What is* $\angle 3$*?*



Problem 1.3. *What is* $\frac{120!}{118!}$ *?*

Problem 1.4. *Find the Euclidean distance between* $(3, 5)$ *and* $(0, 9)$.

Problem 1.5. *Find the taxicab distance between* $(-2, 7)$ *and* $(5, 3)$.

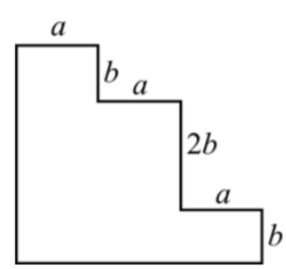
Problem 1.6. *Write* 356.2×10^{-4} *into correct scientific notation.*

Problem 1.7. *In order to sew together three short strips of cloth to get one long strip Cathy needs 18 minutes. How much time does she need to sew together a really long piece consisting of six short strips?*

Problem 1.8. *An item is made from lead blanks in a lathe shop. Each blank suffices for 1 item. Lead shavings accumulated from making 6 items can be melted and made into a blank. How many items can be made from 36 blanks?*

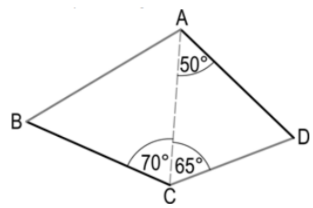
2 20 Points Problems

Problem 2.1. *Suppose* $a = 3$ *and* $b = 2$. *What is the perimeter of the following figure?*



Problem 2.2. *For the positive whole numbers* x, y, z, *the following is true:* $x \times y = 14$, $y \times z = 10$ *and* $z \times x = 35$. *What is the value of* $x + y + z$?

Problem 2.3. *In the diagram below, $AD = BC$. Find $\angle ABC$.*



Problem 2.4. *Suppose Player 1 (you) can choose action A and B. Player 2 (your opponent) can choose action C and D. The payoff (score) is specified in the following table:*

Actions	*C*	*D*
A	*(1, 1)*	*(10, 0)*
B	*(0, 10)*	*(2, 2)*

Find a Nash equilibrium.

Problem 2.5. *Describe (or draw) all possible shapes of a taxicab ellipse.*

Problem 2.6. *Define the function*

$$f(n) = \begin{cases} 5 & \textit{if } n = 0 \\ 2 + 2 \times f(n-1) & \textit{if } n > 0 \end{cases}.$$

What is $f(6)$?

Problem 2.7. *Four cousins are 3, 8, 12 and 14 years old. Emma is younger than Rita. The sum of the ages of Zita and Emma is divisible by 5, as is the sum of the ages of Zita and Rita. How old is Ina (the 4th cousin)?*

Problem 2.8. *If you measure the angles of a triangle, you obtain three different natural numbers. What is the smallest possible sum of the biggest and the smallest angle of the triangle?*

Problem 2.9. *A boy presses a side of a blue pencil to a side of a yellow pencil, holding both pencils vertically. One inch of the pressed side of the blue pencil, measuring from its lower end, is smeared with paint. The yellow pencil is held steady while the boy slides the blue pencil down 1 inch, continuing to press it against the yellow one. He returns the blue pencil to its former position, then again slides it down 1 inch. He continues until he has lowered the blue pencil 5 times and raised it 5 times–10 moves in all. Suppose that during this time the paint neither dries nor diminished in quantity. How many inches of each pencil will be smeared with paint after the tenth move?*

Problem 2.10. *Mark plays a computer game in a 4×4 table. The cells each have a colour which is initially hidden. If he clicks into a cell it changes to red or blue. He knows that there are exactly two blue fields and that they share one side line. Which is the smallest number of clicks with which he can definitely find the blue cells? (Both blue cells have to be clicked.)*

Problem 2.11. *Felix the Tomcat catches 12 fish in 3 days. On the second day he catches more than on the first. On the third day he catches more than on the second but less than on the first two days together. How many fish does he catch on day three?*

3 50 Points Problems

Problem 3.1. *To players pick up 1 through 8 of 101 matches until they are all picked up. You are the first player. To win, you must have an odd number of matches at the end. How do you win this game?*

Problem 3.2. *Anna has five boxes, as well as five black balls and five white balls. She is allowed to decide how she shares out the balls between the boxes as long as she puts at least one ball into each box. Beate randomly chooses one box and takes one ball without looking. Beate wins if she draws a white ball. Otherwise Anna wins. How should Anna distribute the balls in order to get the highest probability of winning?*

Problem 3.3. *2017 people live on an island. Each person is either a liar (who always lies) or a nobleman (who always tells the truth). Over a thousand of them attend a banquet where they all sit together around one big round table. Everyone is saying, "Of my two neighbours, one is a liar and one is a nobleman." What is the maximum number of noblemen on the island?*

Problem 3.4. *On a graphing paper, draw the set*

$$\{P \in \mathbb{R}^2 \ : \ d_T(P, A) - d_T(P, B) = 3\}$$

where $A = (-2, -1)$ *and* $B = (3, 2)$.

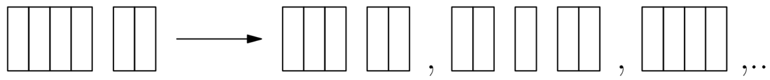
Problem 3.5. *Twenty different positive whole numbers are written into a* 4×5 *table. Two numbers in cells that have one common sideline, always have a common factor greater than 1. Determine the smallest possible value of* n*, if* n *is to be the biggest number in the table.*

Problem 3.6. *Louise draws a line* DE *of length 4 cm. How many ways are there for her to add a point* F *so that a right angled triangle* DEF *with area 1* cm^2 *can be formed?*

Problem 3.7. *At the end of summer camp, the children decided to free the 20 birds they had caught. The Counselor suggested: "Line up the cages in a row. Counting from left to right, open each fifth cage with a bird in it. When you reach the end of the row, start over. You can take the last 2 birds back to the city." You and your friend have your hearts set on a siskin and a thrush. What cages must you place them in to ensure that you can take them with you?*

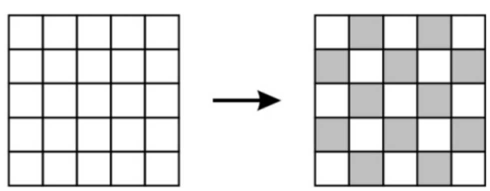
4 100 Points Problems

Problem 4.1. *Arjun and Beth play a game in which they take turns removing one brick or two adjacent bricks from one "wall" among a set of several walls of bricks, with gaps possibly creating new walls. The walls are one brick tall. For example, a set of* *walls of sizes* 4 *and* 2 *can be changed into any of the following by one move:* $(3,2),(2,1,2),(4),(4,1),(2,2)$, *or* $(1,1,2)$.



Arjun plays first, and the player who removes the last brick wins. Write a starting configuration of the walls, containing 3 disconnected walls and 9 bricks in total, such that there is a strategy that guarantees a win for Beth. (For example, $(7,1,1)$ *would be a configuration of 3 walls and 9 bricks, but the* $(4,2)$ *above only have 2 walls and 6 bricks).*

Problem 4.2. *We consider a* 5×5 *square that is split up into 25 fields. Initially all fields are white. In each move it is allowed to change the colour of three fields that are adjacent in a horizontal or vertical line (i.e. white fields turn black and black ones turn white). What is the smallest number of moves needed to obtain the chessboard colouring shown in the diagram?*



Problem 4.3. *How many rectangles with integer length sides have perimeter equal to area? (Rotated rectangles, like* 1×2 *and* 2×1, *are considered the same rectangle)*