Review Game: Combinatorics and Probability March 10, 2024

(1 point) Jamal has a drawer containing 6 green socks, 18 purple socks, and 12 orange socks. After adding more purple socks, Jamal noticed that there is now a 60 percent chance that a sock randomly selected from the drawer is purple. How many purple socks did Jamal add?

(2 points) The numbers from 1 to 49 are arranged in a spiral pattern on a square grid, beginning at the center. The first few numbers have been entered into the grid below. Consider the four numbers that will appear in the shaded squares, on the same diagonal as the number 7. How many of these four numbers are prime?

				5	4	3			
				6	1	2			
				7					
(A) 0	(B) 1	(C) 2						(D) 3	(E) 4

(3 points) In how many ways can the letters in BEEKEEPER be rearranged so that two or more Es do not appear together?

(A) 1 (B) 4 (C) 12 (D) 24 (E) 120

(4 points) Lola, Lolo, Tiya, and Tiyo participated in a ping pong tournament. Each player competed against each of the other three players exactly twice. Shown below are the win-loss records for the players. The numbers 1 and 0 represent a win or loss, respectively. For example, Lola won five matches and lost the fourth match. What was Tiyo's win-loss record?

		Player	Result	_	
		$_{\rm Lola}$	111011	-	
		Lolo	101010		
		Tiya	010100 ??????		
		Tiyo	??????		
(A) 000101	(B) 001001	(C) ()10000	(D) 010101	(E) 011000

(5 points) Alina writes the numbers (1,2, ..., 9) on separate cards, one number per card. She wishes to divide the cards into 3 groups of 3 cards so that the sum of the numbers in each group will be the same. In how many ways can this be done?

$$(A) 0 (B) 1 (C) 2 (D) 3 (E) 4$$

(6 points) A positive integer divisor of 12! is chosen at random. The probability that the divisor chosen is a perfect square can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is m + n?

$$(A) 3 (B) 5 (C) 12 (D) 18 (E) 23$$

(7 points) Debra flips a fair coin repeatedly, keeping track of how many heads and how many tails she has seen in total, until she gets either two heads in a row or two tails in a row, at which point she stops flipping. What is the probability that she gets two heads in a row but she sees a second tail before she sees a second head?

(A)
$$\frac{1}{36}$$
 (B) $\frac{1}{24}$ (C) $\frac{1}{18}$ (D) $\frac{1}{12}$ (E) $\frac{1}{6}$