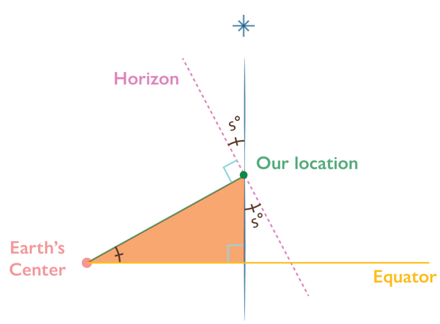
Exploration Circle: Astronomy and Navigation, Notes and Solutions

Curtis Liu & Sunny Liang

March 2024

1. Mt. Everest

2. Assume Polaris is infinitely far away. Angle above horizon of Polaris and latitude are the same number. See diagram.



UCLA Math Sci is at 34.069743279872135, -118.44247789467697

3. Put a stick vertically in the ground, use your compass to draw a North-South line in the ground starting from the stick. Solar noon occurs when the shadow passes over the North-South line.

4. Of course, you should get the same latitude as in Problem 2.

   latitude $= 90°-$ sextant reading at local noon $+$ solar declination

   solar declination $= -23.5° \cdot \cos(360° \cdot \frac{\text{days since winter solstice}}{\text{days in the year}})$

5. (c) distance from subsolar point in NM $= 60 \cdot (90° -$solar altitude angle$)$

   More generally:

   distance from subsolar point in x units
   $=$ (circumference of the earth in x units) $\cdot \frac{(90° - \text{solar altitude angle})}{360°}$

(e) Draw circles centered at each subsolar point with radii corresponding to your calculated distance. See footnote for tips on scaling. The intersection(s) between the two circles are your possible locations.

(f) Two intersecting circles will usually have two points of intersection. Using this method, we will typically find that we have one intersect in the northern hemisphere and one in the southern hemisphere. Knowing which hemisphere is pretty easy though. At night, you can identify constellations. During the day, assuming you're not too close to the equator, you can check the position of the sun relative to a compass. The sun tends to be in the south in the northern hemisphere and north in the southern hemisphere. This effect is exacerbated in the winter and mitigated in the summer.

6. The earth rotates 180° in 12 hours. If your solar noon occurs at the same time as it's 19:52 for somebody in Greenwich, then you are 7.8667 hours behind of latitude 0°. $\frac{7.8667}{12} \cdot 180° = 118°$. Since you are behind Greenwich and the sun rises in the East, you must be 118° west of Greenwich.

7. see infra 9.

8. see infra 9.

9. `https://personal.math.ubc.ca/~israel/m103/mercator/mercator.html`

10. Notice that $\Omega_z = \Omega \cdot \cos(90° - \phi) = \Omega \cdot \sin(\phi)$.

    ϕ is latitude.

    Ω_z is the magnitude of the component of the Earth's angular velocity that is parallel to the pendulum's equilibrium position.

    Ω is the Earth's angular speed, namely 360°/24 hr.

    We know that $\Omega_z = 360°/T$ where T is the period of rotation in hours.

    Rearranging, we get that $T = \frac{360°}{\frac{360°}{24\text{hr}} \sin\phi} = \frac{24\text{hr}}{\sin\phi}$.

    Ask me to draw a diagram if this doesn't make sense.

11. $T = \frac{24}{\sin 34°} \approx 43$ hr

    Actually, Griffith Observatory's pendulum has a period of about 42 hours. I'm not certain but here are a few reasons that might explain the disparity:

    - The earth is not actually spherical. In fact, there are three different definitions of latitude that all vary slightly.
    - The period of Earth's rotation is actually closer to 23 hours, 56 minutes and 4 seconds[1].

[1] You may be tempted to consider the difference between sidereal and solar days. However, the difference is only actually 8.4 ms. Lots of online sources state that solar days are exactly 24 hours, so 4 minutes longer than sidereal days. This is wrong since the hour was redefined according to the atomic second and not the rotation of the earth. In general, the Earth's rotation varies and *leap seconds* have been used to adjust, though leap seconds are now in the process of being abandoned. I'm going to choose to end this rabbit hole here, I encourage you to explore further.

- Griffith Observatory is actually at 34.1184340449039 N. So a bit farther north than UCLA Math Sci's 34.069743279872135 N. I rounded down to 34 anyways which doesn't help.
- Doing physics in rotating reference frames is weird and unintuitive. My above solution makes a lot of hidden assumptions and approximations.

Here are some more detailed analyses that I haven't read:

`https://warwick.ac.uk/fac/sci/physics/intranet/pendulum/derivation/fouc-pend.pdf`

`http://www.cleonis.nl/physics/phys256/foucault_pendulum.php`

12. 377.

13-16. Have fun ☺