1 Team Round

1. Let N be a 6-digit number formed by the digits $1, 2, 3, 3, 4, 5$. Compute the smalelst value of N that is divisible by 264.

2. In triangle ABC, $AB = 4$, $BC = 6$, and $AC = 8$. Squares $ABQR$ and $BCST$ are drawn external to and lie in the same plane as $\triangle ABC$. Compute QT.

3. The numbers $1, 2, \ldots, 8$ are placed in a 3×3 square grid, leaving exactly one blank square. Such a placement is called *okay* if in every pair of adjacent squares, either one square is blank or the difference between the two numbers is at most 2 (two squares are considered adjacent if they share a common side). If reflections, rotations, etc. of placements are considered distinct, compute the number of distinct okay placements.

4. The six sides of convex hexagon $A_1A_2A_3A_4A_5A_6$ are colored red. Each of the diagonals of the hexagon is colored either red or blue. Compute the number of colorings such that every triangle $A_iA_jA_k$ has at least one red side.

5. Let $A_1A_2A_3A_4A_5A_6A_7A_8$ be a regular octagon. Let u be the vector from A_1 to A_2 and let v be the vector from A_1 to A_8. The vector from A_1 to A_4 can be written as $au + bv$ for a unique ordered pair of real numbers (a, b). Compute (a, b).

6. Given an arbitrary finite sequence of letters (represented as a word), a subsequence is a sequence of one or more letters that appear in the same order as in the original sequence. For example, N, CT, OT T, and $CONTEST$ are subsequences of the word $CONTEST$, but NOT, $ONSET$, and $TESS$ are not. Assuming the standard English alphabet $\{A, B, ..., Z\}$, compute the number of distinct four-letter "words" for which EE is a subsequence.

7. The function f satisfies the relation $f(n) = f(n-1)f(n-2)$ for all integers n, and $f(n) > 0$ for all positive integers n. If $f(1) = \frac{f(2)}{512}$ and $\frac{1}{f(1)} = 2f(2)$, compute $f(f(4))$.

8. Compute the area of the region defined by $x^2 + y^2 \leq |x| + |y|$.

9. The equations $x^3 + Ax + 10 = 0$ and $x^3 + Bx^2 + 50 = 0$ have two roots in common. Compute the product of these common roots.

10. Points A and L lie outside circle ω, whose center is O, and $\overline{AL}$ contains diameter $\overline{RM}$, as shown below. Circle ω is tangent to $\overline{LK}$ at K. Also, $\overline{AK}$ intersects ω at Y, which is between A and K. If $KL = 3$, $ML = 2$, and $m\angle AKL - m\angle YMK = 90°$, compute $[AKM]$ (i.e., the area of $\triangle AKM$).