1 Team Round

1. Let \( N \) be a 6-digit number formed by the digits 1, 2, 3, 3, 4, 5. Compute the smallest value of \( N \) that is divisible by 264.

2. In triangle \( ABC \), \( AB = 4 \), \( BC = 6 \), and \( AC = 8 \). Squares \( ABQR \) and \( BCST \) are drawn external to and lie in the same plane as \( \triangle ABC \). Compute \( QT \).

3. The numbers 1, 2, \ldots, 8 are placed in a 3 \times 3 square grid, leaving exactly one blank square. Such a placement is called okay if in every pair of adjacent squares, either one square is blank or the difference between the two numbers is at most 2 (two squares are considered adjacent if they share a common side). If reflections, rotations, etc. of placements are considered distinct, compute the number of distinct okay placements.

4. The six sides of convex hexagon \( A_1A_2A_3A_4A_5A_6 \) are colored red. Each of the diagonals of the hexagon is colored either red or blue. Compute the number of colorings such that every triangle \( A_iA_jA_k \) has at least one red side.

5. Let \( A_1A_2A_3A_4A_5A_6A_7A_8 \) be a regular octagon. Let \( u \) be the vector from \( A_1 \) to \( A_2 \) and let \( v \) be the vector from \( A_1 \) to \( A_8 \). The vector from \( A_1 \) to \( A_4 \) can be written as \( au + bv \) for a unique ordered pair of real numbers \((a, b)\). Compute \((a, b)\).

6. Given an arbitrary finite sequence of letters (represented as a word), a subsequence is a sequence of one or more letters that appear in the same order as in the original sequence. For example, \( N \), \( CT \), \( OT \), and \( CONTEST \) are subsequences of the word \( CONTEST \), but \( NOT \), \( ONSET \), and \( TESS \) are not. Assuming the standard English alphabet \{A, B, ..., Z\}, compute the number of distinct four-letter “words” for which \( EE \) is a subsequence.

7. The function \( f \) satisfies the relation \( f(n) = f(n-1)f(n-2) \) for all integers \( n \), and \( f(n) > 0 \) for all positive integers \( n \). If \( f(1) = \frac{5}{12} \) and \( \frac{1}{f(1)} = 2f(2) \), compute \( f(f(4)) \).

8. Compute the area of the region defined by \( x^2 + y^2 \leq |x| + |y| \).

9. The equations \( x^3 + Ax + 10 = 0 \) and \( x^3 + Bx^2 + 50 = 0 \) have two roots in common. Compute the product of these common roots.

10. Points \( A \) and \( L \) lie outside circle \( \omega \), whose center is \( O \), and \( \overline{AL} \) contains diameter \( \overline{RM} \), as shown below. Circle \( \omega \) is tangent to \( \overline{LK} \) at \( K \). Also, \( \overline{AK} \) intersects \( \omega \) at \( Y \), which is between \( A \) and \( K \). If \( KL = 3 \), \( ML = 2 \), and \( m \angle AKL - m \angle YMK = 90^\circ \), compute \( [AKM] \) (i.e., the area of \( \triangle AKM \)).