3 Individual Round

1. For each positive integer k, let S_k denote the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is k. For example, S_3 is the sequence $1, 4, 7, 10, \ldots$. For how many values of k does S_k contain the term 2005?

2. There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

3. Pat wants to buy four donuts from an ample supply of three types of donuts: glazed, chocolate, and powdered. How many different selections are possible?

   (A) 6 **(B)** 9 **(C)** 12 **(D)** 15 **(E)** 18

4. An integer N is selected at random in the range $1 \leq N \leq 2020$. What is the probability that the remainder when N^{16} is divided by 5 is 1?

   (A) $\frac{1}{5}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{4}{5}$ **(E)** 1

5. Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90°$. Let M and N be the midpoints of legs OX and OY, respectively. Given that $XN = 19$ and $YM = 22$, find XY.

   (A) 24 (B) 26 (C) 28 (D) 30 (E) 32

6. In the expansion of

$$\left(1 + x + x^2 + \cdots + x^{27}\right)\left(1 + x + x^2 + \cdots + x^{14}\right)^2,$$

   what is the coefficient of x^{28}?

   (A) 195 (B) 196 (C) 224 (D) 378 (E) 405

7. A function f is defined recursively by $f(1) = f(2) = 1$ and

$$f(n) = f(n-1) - f(n-2) + n$$

for all integers $n \geq 3$. What is $f(2018)$?

(A) 2016 **(B)** 2017 **(C)** 2018 **(D)** 2019 **(E)** 2020

8. Triangle ABC has $AB = 13, BC = 14$ and $AC = 15$. Let P be the point on $\overline{AC}$ such that $PC = 10$. There are exactly two points D and E on line BP such that quadrilaterals $ABCD$ and $ABCE$ are trapezoids. What is the distance DE?

(A) $\frac{42}{5}$ **(B)** $6\sqrt{2}$ **(C)** $\frac{84}{5}$ **(D)** $12\sqrt{2}$ **(E)** 18

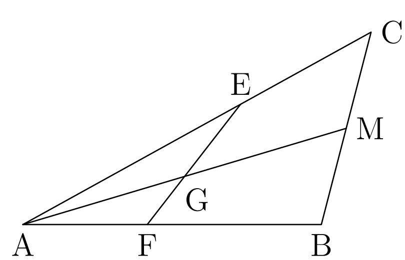
9. How many ordered pairs (a, b) of positive integers satisfy the equation

$$a \cdot b + 63 = 20 \cdot \text{lcm}(a, b) + 12 \cdot \text{gcd}(a, b),$$

where $\text{gcd}(a, b)$ denotes the greatest common divisor of a and b, and $\text{lcm}(a, b)$ denotes their least common multiple?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

10. In $\triangle ABC$ shown in the adjoining figure, M is the midpoint of side BC, $AB = 12$ and $AC = 16$. Points E and F are taken on AC and AB, respectively, and lines EF and AM intersect at G. If $AE = 2AF$ then $\frac{EG}{GF}$ equals



(A) $\frac{3}{2}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{5}{4}$ **(D)** $\frac{6}{5}$ **(E)** not enough information to solve the problem