OLGA RADKO MATH CIRCLE: ADVANCED 3

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Winter Final Exam II

Name: ____

Problem 1	/10
Problem 2	/10
Problem 3	/10
Problem 4	/10
Total	/40

Let V be a vector space over the field $\mathbb{F}$. We say that a set $S \subseteq V$ is *linearly independent* over $\mathbb{F}$ if, whenever $a_1 s_1 + ... + a_n s_n = 0$ for some scalars a_i in $\mathbb{F}$, we must have all $a_i = 0$.

Problem II.1:

(1) Consider $V = \mathbb{C}$ and $\mathbb{F} = \mathbb{R}$. Is the set $S = \{1, i\}$ linearly independent over $\mathbb{R}$?
(2) Consider $V = \mathbb{C}$ and $\mathbb{F} = \mathbb{C}$. Is the set $S = \{1, i\}$ linearly independent over $\mathbb{C}$?

Solution:

Consider the vector space $V = \mathbb{A}^2_{\mathbb{F}_3}$ over the field $\mathbb{F}_3$. Consider $(1, 2)$ in V.

Problem II.2:

Find, with a proof, a vector v in V such that the set $\{(1, 2), v\}$ is a basis for V over $\mathbb{F}_3$.

For this problem you may use without proof that a set S is a basis if and only if it is a generating set and it is linearly independent.

Solution:

Consider $V = \mathbb{A}^3_{\mathbb{F}_3}$ over the field $\mathbb{F}_3$.

Problem II.3:

(1) What is the largest size of a set $S \subseteq V$, such that any two elements of S are linearly independent?
(2) What is the largest size of a set $S \subseteq V$, such that any three elements of S are linearly independent?
(3) What is the largest size of a linearly independent set $S \subseteq V$?

Solution:

Problem II.4:

(1) In how many points can two lines intersect in $\mathbb{A}^2_{\mathbb{F}_q}$?
(2) In how many points can two hyperplanes intersect in $\mathbb{A}^n_{\mathbb{F}_q}$?
(3) In how many points can three hyperplanes intersect in $\mathbb{A}^n_{\mathbb{F}_q}$?

Solution:

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