OLGA RADKO MATH CIRCLE: ADVANCED 3

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Winter Final Exam I

Name: ____

Problem 1	/10
Problem 2	/10
Problem 3	/10
Problem 4	/10
Total	/40

Let $\mathbb{P}^2_{\mathbb{F}_3}$ be the projective plane over $\mathbb{F}_3$.

Problem I.1 :

(1) Which of the following coordinates represent the same point in $\mathbb{P}^2_{\mathbb{F}_3}$:

$$\{[1:1:1],[1:1:2],[1:2:1],[1:2:2],[2:1:1],[2:1:2],[2:2:1],[2:2:2]\}$$

(2) List all the points in $\mathbb{P}^3_{\mathbb{F}_3}$.

Solution:

Inside a field $\mathbb{F}$, an element a is called a square if there exists some x in $\mathbb{F}$ such that $a = x^2$.

Problem I.2:

(1) Show that any element in $\mathbb{F}_7$ is the sum of two squares.
(2) Show that for any prime number p, any element in $\mathbb{F}_p$ is the sum of two squares.
(3) Is any element in $\mathbb{F}_q$ a sum of two squares?

Remark: Remember that q may not be a prime number.

Solution:

In a field any nonzero element has a multiplicative inverse, therefore we can define division by nonzero elements.

Problem I.3:

Consider the expression:

$$f(\alpha) = \frac{\alpha - 2}{\alpha^2 - 5\alpha - 1}$$

(1) What possible values can $f(\alpha)$ have when α takes values in $\mathbb{F}_7$ different from 2 and 3.
(2) What possible values can $f(\alpha)$ have when α takes values in $\mathbb{F}_{49}$ different from 2 and 3.
(3) What possible values can $f(\alpha)$ have when α takes values in $\mathbb{F}_{7^k}$ different from 2 and 3.

Solution:

For any prime p, any integer x yields an element of $\mathbb{F}_p$, this can be done by taking the remainder after dividing x by p.

Problem I.4:

(1) Show that any point of $\mathbb{P}^2_{\mathbb{F}_3}$ can be written as $[x : y^2 : z^3]$, where x, y, z are integers.
(2) Show that some points of $\mathbb{P}^3_{\mathbb{F}_3}$ cannot be written as $[x : y^2 : z^3 : w^4]$, where x, y, z, w are integers.
(3) Show that for any odd prime number p, some points of $\mathbb{P}^3_{\mathbb{F}_p}$ cannot be written as $[x : y^2 : z^3 : w^4]$, where x, y, z, w are integers.

Solution:

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