Game Theory

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1.1 First player wins.

Put a chip at the center, then mirror the other player's move.

1.2 If there are only two people,

$$I > J$$

I gets all coins and J gets 0.

If there are 3 people,

$$H > I > J$$

J would vote yes as soon as he gets more than 0. So H gets 999, I gets 0, J gets 1.

If 4 people,

$$G > H > I > J$$

I would vote yes if getting more than 0, So G gets 999, I gets 1, H, J gets 0.

Continue the pattern, A would propose

A: 996, B: 0, C: 1, D: 0

E: 1, F: 0, G: 1, H: 0

I: 1, J: 0

And A, C, E, G, I vote yes.

1.3 If there are only two people,

$$I > J$$

I gets all coins and J gets 0.

If there are 3 people,

$$H > I > J$$

J would vote yes as soon as he gets at least 0. So H gets 1000, I gets 0, J gets 0.

Continue the pattern. A would take all 1000 coins, and everyone except B would agree.

2.1 For United:

\$200 is dominant strategy

For American Airline:

\$200 is dominant strategy

Result: both airlines sell tickets at \$200, both got profits - \$10.

3.1

(B, C) is the only Nash Equilibrium

3.2 Yes.

	C	D
A	(2, 2)	(0, 1)
B	(1, 3)	(1, 4)

Both (A, C) (B, D) are Nash Equilibrium

3.3 Rock Paper Scissors

3.5 An outcome where Player 2 is minimizing the payoff along the column but maximizing along the row.

3.6 (B, E) is a saddle point and a Nash equilibrium

3.7 Fix Player 2's choice, then the saddle point is where Player 1 maximizes the payoff.

Fix Player 1's choice, then at the saddle point, Player 1 has the minimal payoff across the row, meaning that Player 2 has the maximal payoff (since this is a zero-sum game)

Thus, a saddle point must be a Nash equilibrium (each player has the maximal payoff given the other players' choices fixed)

3.8 D is a dominant strategy for Player 2, so Player 2 chooses D.

Player 1 chooses C to maximize payoff.

4.1 Payoff for L:

$$0.1 \times 3 + 0.9 \times 1 = 1.2$$

Payoff for R:

$$0.1 \times 0 + 0.9 \times 2 = 1.8$$

Since $1.8 > 1.2$, player 2 will choose R all the time. (Pure strategy)

Player 1's payoff:

$$0.1 \times 3 + 0.9 \times 1 = 1.2$$

4.2

Player 1's payoff:

$$0 p_1 q_1 + 2 p_2 q_1 + 3 p_1 q_2 + 1 p_2 q_2$$
$$= 2 p_2 q_1 + 3 p_1 q_2 + p_2 q_2$$

Player 2's payoff:

$$3 p_1 q_1 + 1 p_2 q_1 + 0 p_1 q_2 + 2 p_2 q_2$$
$$= 3 p_1 q_1 + p_2 q_1 + 2 p_2 q_2$$

4.3 Use the results in 4.2.

If $S_2 = (1, 0)$, player 1's payoff is

$$2 p_2 q_1 + 3 p_1 q_2 + p_2 q_2$$
$$= 2 p_2 \cdot 1 + 3 p_1 \cdot 0 + p_2 \cdot 0$$
$$= 2 p_2$$

If $S_2 = (0, 1)$,

$$2 p_2 q_1 + 3 p_1 q_2 + p_2 q_2$$
$$= 2 p_2 \cdot 0 + 3 p_1 \cdot 1 + p_2 \cdot 1$$
$$= 3 p_1 + p_2$$
$$= 2 p_1 + 1 \qquad (\text{since } p_1 + p_2 = 1)$$
$$= 3 - 2 p_2$$

So $\min\{2p_2, 3 - 2p_2\}$ is maximized at

$$p_1 = \frac{1}{4}, \quad p_2 = \frac{3}{4}$$

4.4 Player 1's optimal strategy is

$$p_1 = \frac{1}{4}, \quad p_2 = \frac{3}{4}$$

(Since this is a constant-sum game, Player 2 will always try to minimize Player 1's payoff, so Player 1's best strategy is to maximize the minimal expected payoff.)

Player 2 payoff:

$$3 p_1 q_1 + p_2 q_1 + 2 p_2 q_2$$
$$= \frac{1}{4} q_1 + \frac{3}{4} q_1 + \frac{3}{2} q_2$$
$$= q_1 + \frac{3}{2}(1 - q_1)$$
$$= \frac{3}{2} - \frac{1}{2} q_1$$

which maximizes at

$$q_1 = 0, \quad q_2 = 1$$

So Player 2 chooses pure strategy S_2'.

4.5 Everything is symmetric, so we should expect

$$p_1 = p_2 = p_3 = q_1 = q_2 = q_3 = \frac{1}{3}$$

4.6 Pure equilibrium: One swerve and one straight

For mixed strategy, Player 1's payoff:

$$0 p_1 q_1 + 1 p_2 q_1 - 1 p_1 q_2 - 4 p_2 q_2$$
$$= p_2 q_1 - p_1 q_2 - 4 p_2 q_2$$
$$= p_2 q_1 - (1 - p_2) q_2 - 4 p_2 q_2$$
$$= p_2 (q_1 - 3 q_2) - q_2$$

So Non-pure strategy occurs only when

$$q_1 - 3 q_2 = 0 \Rightarrow q_1 = 0.75, \ q_2 = 0.25$$

By symmetry, player 2 will only choose non-pure strategy when

$$p_1 = 0.75, \quad p_2 = 0.25$$

So the only mixed NE is

$$(p_1, p_2) = (q_1, q_2) = (0.75, 0.25)$$

Both have payoff

$$-0.25$$

4.8 Each player pick a natural number, and whoever has the bigger number wins.