Big Numbers

Sunday, February 18, 2024 06:28

1.1 9^{9^9}

1.2 $2^{2^{22}}$

1.3 $2^{400} = 4^{200} > 3^{200}$

2.1 2×10^7

5.6209×10^{13}

3.2×10^{-6}

5.24×10^9

3.2×10^{10}

2.2 5.3×10^6

-2.042×10^8

1.1×10^{19}

1.6667×10^{-2}

6.5536×10^4

2.3 7.5

2.4

$10^{10^{100}}$

2.5 1.06×10^{84}

2.6

$$2^{60} = 16^{15} = 16 \times 16^{14} > 16 \times 10^{14} = 1.6 \times 10^{15}$$

So the thickness of paper folded 60 times is:

$$0.1 \times \frac{1}{1000} \times 2^{60} > 0.1 \times \frac{1}{1000} \times 1.6 \times 10^{15} = 1.6 \times 10^{11} \text{ meters}$$

$$149 \text{ million km} = 149 \times 1000000 \times 1000 = 1.49 \times 10^{11} \text{ meters} < 1.6 \times 10^{11} \text{ meters}$$

So paper folded 60 times is thicker.

3.1

(1)

$$\frac{10!}{3^{10}} = \frac{10 \times 9 \times 8 \times \cdots \times 3 \times 2 \times 1}{3 \times 3 \times 3 \times \cdots \times 3 \times 3 \times 3}$$

$$= \underbrace{\frac{10 \times 9 \times 2 \times 1}{3 \times 3 \times 3 \times 3}}_{>1} \times \underbrace{\frac{8 \times 7 \times \cdots \times 3}{3 \times 3 \times \cdots \times 3}}_{>1}$$

So $10! > 3^{10}$

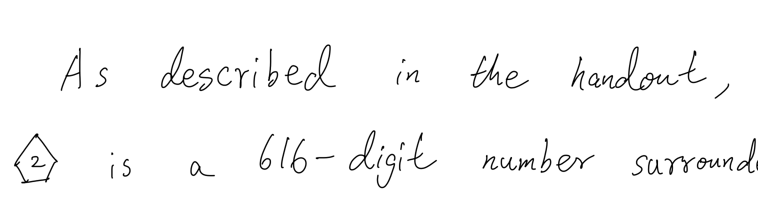
(2)

$$\frac{100!}{10^{100}} = \frac{100 \times 99 \times \cdots \times 10 \times 9 \times \cdots \times 1}{10 \times 10 \times \cdots \times 10 \times 10 \times \cdots \times 10} > 1$$

So $100! > 10^{100}$

(3) (Googol)! > Googolplex

3.2



$$= \left(3^{3^4}\right)^{3^{3^4}} = 3^{3^{4+3^4}} = 3^{3^{85}}$$

3.3 1

3.4 As described in the handout, 2 is a 616-digit number surrounded by 254 triangles. Getting rid of the inner-most triangle, we get a number x^x surrounded by 253 triangles, where x has 616 digits. This is already larger than $10^{10^{100}}$. So

2 > A Googolplex

3.5 n is n surrounded by n pentagons.

n is n surrounded by n hexagons.

3.6

n surrounded by an n-gon is defined as n surrounded by n (n−1)-gons.

3.7

Moser number is much bigger. In fact, the solution for 3.4 already implies

2 > $\text{Googolplex}^{\text{Googolplex}}$

And the Moser number is much bigger than 2.

3.8

An example would be to define

$$n * 1 := \underbrace{(\cdots((n!)!)! \cdots)!}_{n \text{ factorials}}$$

for every $m > 1$, define

$$n * m = \underbrace{(\cdots((n * (m-1)) * (m-1)) * \cdots * (m-1))}_{n \cdot (n * (m-1)) \text{ copies of } m-1}$$

Then consider

$$9 * (9 * 9)$$

4.1 $F(4) = 4 + F(3) = 10$

$F(5) = 5 + F(4) = 15$

$F(6) = 6 + F(5) = 21$

$F(100) = 1 + 2 + \cdots + 100 = 5050$

4.2

n	1	2	3	4	5	6	7	8	9	10
G(n)	1	2	6	12	23	41	71	120	200	330

4.3

$$F(n) = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot F(n-1) & \text{if } n > 0 \end{cases}$$

4.4

$m \backslash n$	0	1	2	3	4	5	6	7	8
0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	5	7	9	11	13	15	17	19
3	5	13	29	61	125	253	509	1021	2045
4	$2^{2^2} - 3$	$2^{2^{2^2}} - 3$	$2^{2^{2^{2^2}}} - 3$	$2^{2^{2^{2^{2^2}}}} - 3$	$2^{2^{2^{2^{2^{2^2}}}}} - 3$	$\cdots$			

4.5 Notice that for $n > 0$,

$$A(0, n) = n + 1$$

$$A(1, n) = A(0, A(1, n-1)) = A(1, n-1) + 1$$

$$\Downarrow$$

$$A(1, n) = n + 2$$

Similarly,

$$A(2, n) = A(1, A(2, n-1)) = A(2, n-1) + 2$$

$$\Downarrow$$

$$A(2, n) = 2n + 3$$

$$A(3, n) = A(2, A(3, n-1)) = 2 \cdot A(3, n-1) + 3$$

$$\Downarrow$$

$$A(3, n) = 2^{n+3} - 3$$

Now let's prove our problem by induction,

If $n = 0$,

$$A(4, 0) = A(3, 1) = 13 = 2^{2^2} - 3$$

For $n > 0$, assume the statement is true for $A(4, n-1)$, then

$$A(4, n) = A(3, A(4, n-1))$$

$$= A(3, 2^{2^{\cdot^{\cdot^{\cdot^2}}}} - 3) \quad (n+2 \text{ copies of } 2)$$

$$= 2^{2^{\cdot^{\cdot^{\cdot^2}}} - 3 + 3} - 3 \quad (n+3 \text{ copies of } 2)$$

$$= 2^{2^{\cdot^{\cdot^{\cdot^2}}}} - 3 \quad (n+3 \text{ copies of } 2)$$

Done by induction.

4.6

$$A(5, 1) = A(4, A(5, 0))$$

$$= A(4, A(4, 1))$$

$$= A(4, 2^{2^{2^2}} - 3)$$

$$= A(4, 65533)$$

$$= 2^{2^{\cdot^{\cdot^{\cdot^2}}}} - 3 \quad (65536 \text{ copies of } 2)$$

4.7

$$\left.2^{2^{\cdot^{\cdot^{\cdot^2}}}}\right\} 65536 \text{ copies of } 2$$