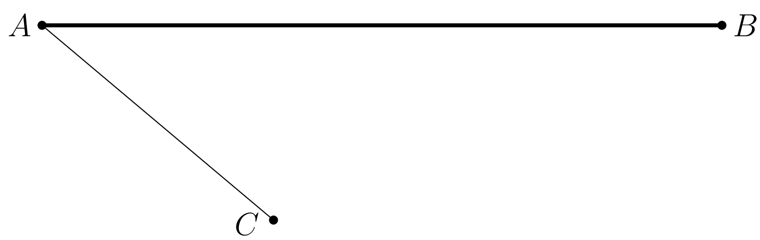
Vectors and Physics 1

Problem 4 Solution

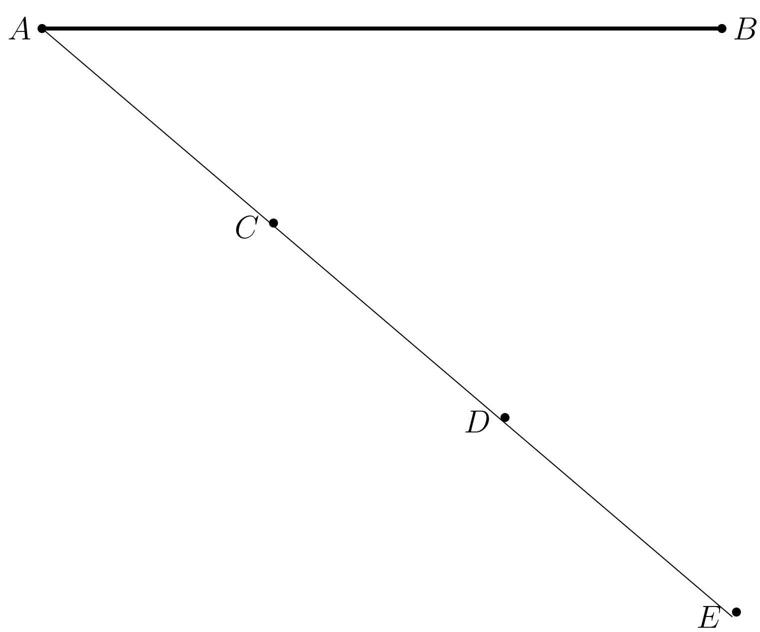
How to divide a segment into parts

How to divide a segment AB into three parts:

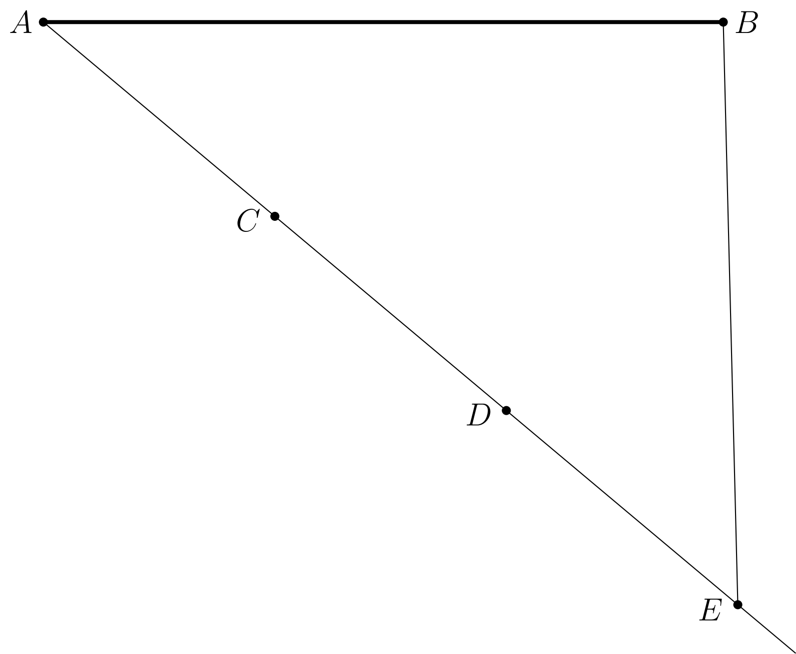
1. Mark an additional segment AC with any length.



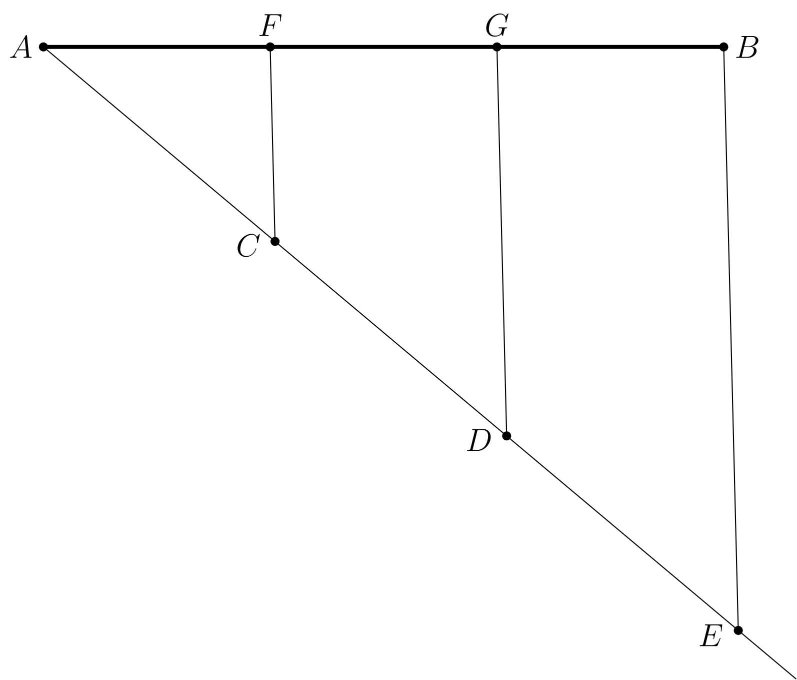
2. Extend AC to create segments CD and DE with the same length. To find point D, draw a circle centered at point C with radius $|AC|$. Repeat with a circle around D to find E.



3. Mark line BE.



4. Mark line CF and GD so that they are parallel to BE.

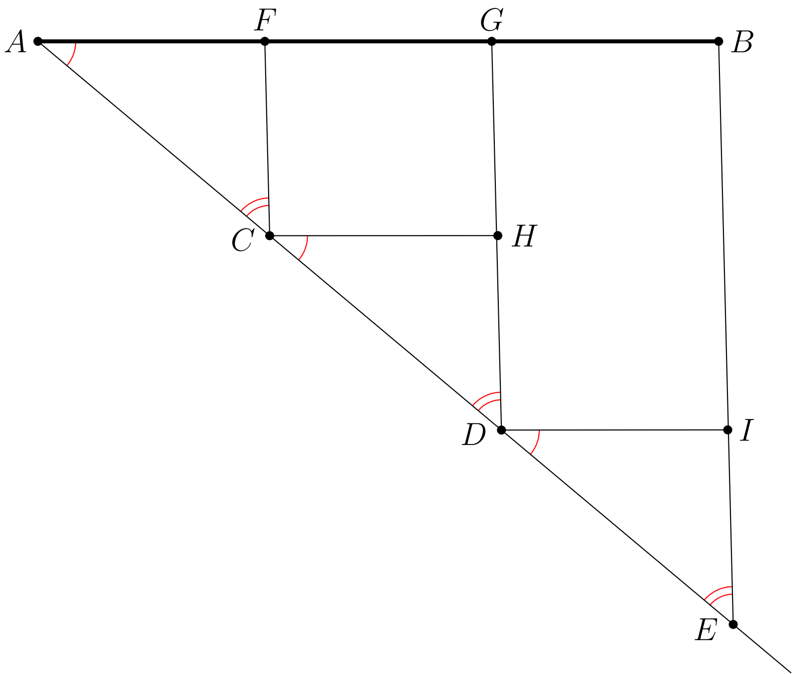


AF, FG, and GB are each one-third of the length of AB.

This construction is extremely useful, as it allows us to divide a segment into any number of equal parts.

Proposition 1 $|AF| = |FG| = |GB|$

Proof — Draw straight lines CH and DI parallel to AB.



Claim	**Reason**
1. $\lvert AC\rvert = \lvert CD\rvert = \lvert DE\rvert$	By construction: These line segments were drawn to be the same length.

Claim	**Reason**
2. $\angle FAC \cong \angle HCD \cong \angle IDE$	Proposition 2 from the 2/18/24 Packet, Intro to Geometry - Parallel Lines These are corresponding angles, since $AF \parallel CH \parallel DI$.
3. $\angle FCA \cong \angle HDC \cong \angle IED$	Proposition 2 from the 2/18/24 Packet, Intro to Geometry - Parallel Lines These are corresponding angles, since $CF \parallel DH \parallel EI$.
4. $\triangle ACF \cong \triangle CDH \cong \triangle DEI$	Theorem 1 from the 1/14/2024 Packet, Intro to Geometry - Angles, Triangles, and Congruence These triangles are congruent by **ASA**, as they share a congruent **A**ngle, **S**ide, and **A**ngle (in that order). We proved that these were congruent in claim 2 (**A**ngle), 1 (**S**ide), and 3 (**A**ngle).
5. $\|AF\| = \|CH\| = \|DI\|$	$\triangle ACF \cong \triangle CDH \cong \triangle DEI$ Corresponding sides of congruent triangles are congruent. These sides are the same side from three congruent triangles.

Claim	**Reason**
6. $CHGF$ and $DIBG$ are parallelograms.	By definition of parallelograms: Because $FG \vert\vert CH$ and $FC \vert\vert GH$, $CHGF$ is a parallelogram. Because $GB \vert\vert DI$ and $GD \vert\vert BI$, $DIBG$ is a parallelogram.
7. $\vert CH\vert = \vert FG\vert$ $\vert DI\vert = \vert GB\vert$	Opposite sides of parallelograms are congruent. $CHGF$ and $DIBG$ are both parallelograms.
8. $\vert AF\vert = \vert FG\vert = \vert GB\vert$	From claim 1, we know that $\vert AF\vert = \vert CH\vert = \vert DI\vert$. Using claim 7, we can substitute $\vert CH\vert$ with $\vert FG\vert$ and $\vert DI\vert$ with $\vert GB\vert$.

Therefore, the three segments AF, FG, and GB are congruent, so we have proved Proposition 1.

Q.E.D. (The proof is complete.)

Note that this proof can be repeated to divide a segment into any integer number of pieces using only a compass and straight edge.