Angles

Sunday, January 14, 2024 3:10 PM

1.1 4

1.2 $x = 65^\circ$

$y = 115^\circ$

$z = 65^\circ$

1.3 a & e , c & g

b & f , d & h

1.4

$a = e = d = h = 135^\circ$

$c = g = b = f = 45^\circ$

1.5

$$\begin{cases} 4y - 12^\circ = 180^\circ \\ x + y = 180^\circ \end{cases} \Rightarrow \begin{cases} x = 132^\circ \\ y = 48^\circ \end{cases}$$

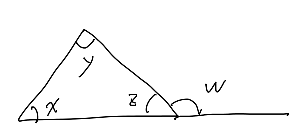
2.1 $\angle BDC = 70^\circ$

$\angle BCD = 180^\circ - 50^\circ - 70^\circ = 60^\circ$

2.2 $7x + 5 + 60 = 20x$

$x = \frac{65}{14}$

2.3



$x + y + z = 180^\circ$

$w + z = 180^\circ$

So $x + y + \cancel{z} = w + \cancel{z}$

$x + y = w$

2.4 In $\triangle CDE$, the interior angles and exterior angles form the relation

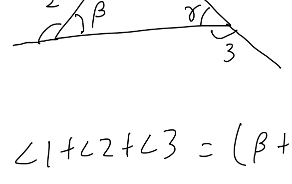
$\angle ECD + \angle CED = \angle DEF$

$30^\circ + (2x - 20^\circ) = x + 55^\circ$

So $x = 45^\circ$,

$\angle CED = 180^\circ - (x + 55^\circ) = 80^\circ$

2.5

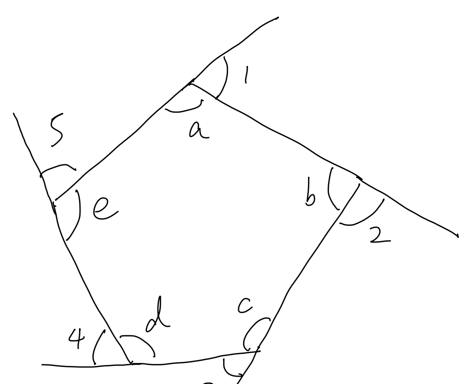


$\angle 1 + \angle 2 + \angle 3 = (\beta + \gamma) + (\alpha + \gamma) + (\alpha + \beta)$

$= 2\alpha + 2\beta + 2\gamma$

$= 360^\circ$.

2.6



$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5$

$= (180^\circ - a) + (180^\circ - b) + \cdots + (180^\circ - e)$

$= 5 \times 180^\circ - (a + b + c + d + e)$

$= 5 \times 180^\circ - \underbrace{3 \times 180^\circ}$

$= 360^\circ$

This can be shown by subdividing the pentagon into 3 triangles

2.7

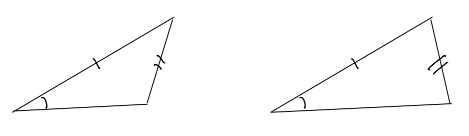
$\frac{360}{1+2+6} = 40^\circ$

If such triangle exists, the exterior angles are 40°, 80°, 240°. But an exterior angle cannot be greater than 180°.

3.1 (a), (b), (f)

3.2 same radius

3.3

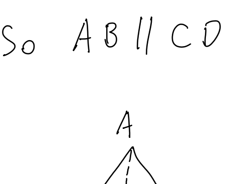


3.4 SAS

Since $\triangle ABE \cong \triangle CDE$, $\angle BAE = \angle DCE$

So $AB \parallel CD$

3.5



$$\begin{cases} \angle ADB = \angle ADC = 90^\circ \\ AB = AC \\ AD = AD \end{cases}$$

3.6

{ All corresponding sides are equal
And
One pair of corresponding angles are equal

Or

{ Three pairs of corresponding sides are equal
And
The two pairs of angles between them are equal

3.7 Since $AE = CF$, $AD = CD$,

We have $ED = FD$

$\triangle ADF \cong \triangle CDE$ by SAS

So $\angle 1 = \angle 2$

3.8 $\triangle ABD \cong \triangle CBD$ by SSS

So $\angle ABD = \angle CBD$

$\triangle ABE \cong \triangle CBE$ by SAS

So $\angle AEB = \angle CEB = \frac{180^\circ}{2} = 90^\circ$

3.9 $\angle EDC + \angle DGA = \angle AFD = 90^\circ$

$\angle EDC + \angle ADF = 90^\circ$

So $\angle DGA = \angle ADF \underset{(AD \parallel BC)}{=} \angle DEC$

So $$\begin{cases} \angle DGA = \angle DEC \\ \angle ADG = \angle DCE \\ AD = DC \end{cases}$$

So $\triangle ADG \cong \triangle DCE$ by AAS

3.10 $\angle EBD = \angle FAD = 45^\circ$

$AD = BD$

Since $\angle ADF + \angle ADE = 90^\circ$

$\angle BDE + \angle ADE = 90^\circ$

$\angle ADF = \angle BDE$

So $\triangle AFD \cong \triangle BED$ by ASA