ORMC AMC 10/12 Group
Winter, Week 5: Review

February 4, 2024

1. **(2012 AMC 10B #4)** When Ringo places his marbles into bags with 6 marbles per bag, he has 4 marbles left over. When Paul does the same with his marbles, he has 3 marbles left over. Ringo and Paul pool their marbles and place them into as many bags as possible, with 6 marbles per bag. How many marbles will be leftover?

2. **(2012 AMC 10A #6)** The product of two positive numbers is 9. The reciprocal of one of these numbers is 4 times the reciprocal of the other number. What is the sum of the two numbers?

3. **(2012 AMC 10A #8)** The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?

4. **(2017 AMC 10A #8)** At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur within the group?

5. **(2012 AMC 10A #9)** A pair of six-sided dice are labeled so that one die has only even numbers (two each of 2, 4, and 6), and the other die has only odd numbers (two of each 1, 3, and 5). The pair of dice is rolled. What is the probability that the sum of the numbers on the tops of the two dice is 7?

6. **(2012 AMC 10B #9)** Two integers have a sum of 26. When two more integers are added to the first two integers the sum is 41. Finally when two more integers are added to the sum of the previous four integers the sum is 57. What is the minimum number of odd integers among the 6 integers?

7. **(2012 AMC 10A #11)** Externally tangent circles with centers at points A and B have radii of lengths 5 and 3, respectively. A line externally tangent to both circles intersects ray AB at point C. What is BC?

8. **(2012 AMC 10B #11)** A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

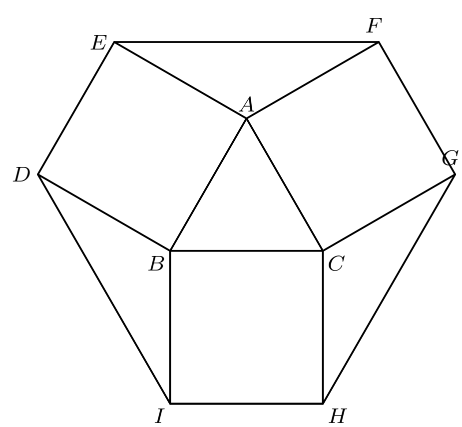
9. **(2018 AMC 10A #11)** When 7 fair standard 6-sided dice are thrown, the probability that the sum of the numbers on the top faces is 10 can be written as

$$\frac{n}{6^7},$$

where n is a positive integer. What is n?

10. **(2012 AMC 10A #12)** A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012, a Tuesday. On what day of the week was Dickens born?

11. **(2014 AMC 10A #13)** Equilateral $\triangle ABC$ has side length 1, and squares $ABDE$, $BCHI$, $CAFG$ lie outside the triangle. What is the area of hexagon $DEFGHI$?



12. **(2017 AMC 10A #13)** Define a sequence recursively by $F_0 = 0,\ F_1 = 1$, and $F_n =$ the remainder when $F_{n-1} + F_{n-2}$ is divided by 3, for all $n \geq 2$. Thus the sequence starts $0, 1, 1, 2, 0, 2, \ldots$ What is $F_{2017} + F_{2018} + F_{2019} + F_{2020} + F_{2021} + F_{2022} + F_{2023} + F_{2024}$?

13. **(2017 AMC 10B #13)** There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes?

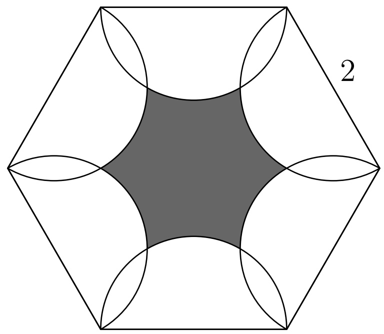
14. **(2019 AMC 10A #13)** Let $\triangle ABC$ be an isosceles triangle with $BC = AC$ and $\angle ACB = 40°$. Construct the circle with diameter $\overline{BC}$, and let D and E be the other intersection points of the circle with the sides $\overline{AC}$ and $\overline{AB}$, respectively. Let F be the intersection of the diagonals of the quadrilateral $BCDE$. What is the degree measure of $\angle BFC$?

15. **(2022 AMC 10B #13)** The positive difference between a pair of primes is equal to 2, and the positive difference between the cubes of the two primes is 31106. What is the sum of the digits of the least prime that is greater than those two primes?

16. **(2012 AMC 10B #14)** Two equilateral triangles are contained in square whose side length is $2\sqrt{3}$. The bases of these triangles are the opposite side of the square, and their intersection is a rhombus. What is the area of the rhombus?

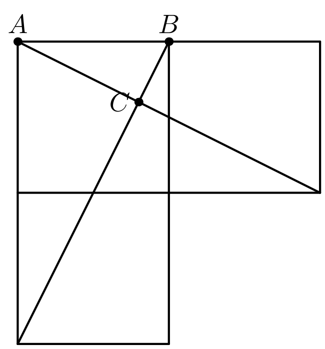
17. **(2013 AMC 10B #14)** Define $a \clubsuit b = a^2b - ab^2$. Which of the following describes the set of points (x, y) for which $x \clubsuit y = y \clubsuit x$?

18. **(2020 AMC 10B #14)** As shown in the figure below, six semicircles lie in the interior of a regular hexagon with side length 2 so that the diameters of the semicircles coincide with the sides of the hexagon. What is the area of the shaded region —- inside the hexagon but outside all of the semicircles?

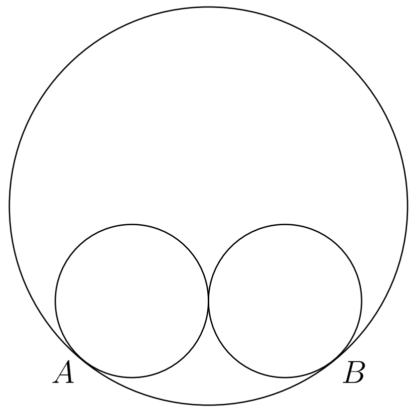


19. **(2020 AMC 12B #10)** In unit square $ABCD$, the inscribed circle ω intersects $\overline{CD}$ at M, and $\overline{AM}$ intersects ω at a point P different from M. What is AP?

20. **(2012 AMC 10A #15)** Three unit squares and two line segments connecting two pairs of vertices are shown. What is the area of $\triangle ABC$?



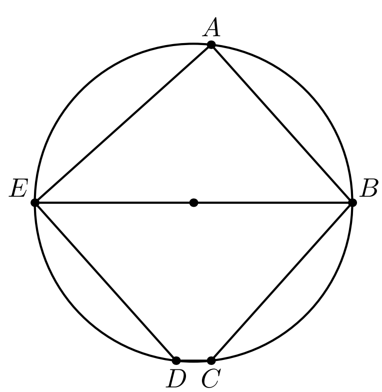
21. **(2018 AMC 10A #15)** Two circles of radius 5 are externally tangent to each other and are internally tangent to a circle of radius 13 at points A and B, as shown in the diagram. The distance AB can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?



22. **(2016 AMC 12A #12)** In $\triangle ABC$, $AB = 6$, $BC = 7$, and $CA = 8$. Point D lies on $\overline{BC}$, and $\overline{AD}$ bisects $\angle BAC$. Point E lies on $\overline{AC}$, and $\overline{BE}$ bisects $\angle ABC$. The bisectors intersect at F. What is the ratio $AF : FD$?

23. **(2020 AMC 12B #12)** Let $\overline{AB}$ be a diameter in a circle of radius $5\sqrt{2}$. Let $\overline{CD}$ be a chord in the circle that intersects $\overline{AB}$ at a point E such that $BE = 2\sqrt{5}$ and $\angle AEC = 45°$. What is $CE^2 + DE^2$?

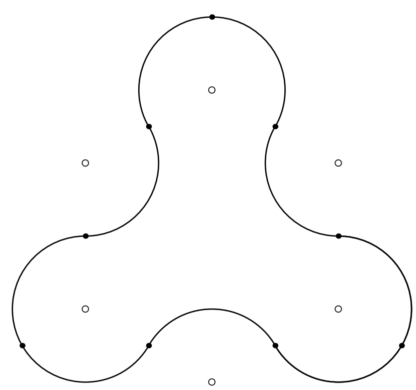
24. **(2011 AMC 10B #17)** In the given circle, the diameter $\overline{EB}$ is parallel to $\overline{DC}$, and $\overline{AB}$ is parallel to $\overline{ED}$. The angles AEB and ABE are in the ratio $4 : 5$. What is the degree measure of angle BCD?



25. **(2012 AMC 10A #17)** Let a and b be relatively prime positive integers with $a > b > 0$ and $\dfrac{a^3 - b^3}{(a-b)^3} = \dfrac{73}{3}$. What is $a - b$?

26. **(2015 AMC 12B #13)** Quadrilateral $ABCD$ is inscribed in a circle with $\angle BAC = 70°, \angle ADB = 40°, AD = 4$, and $BC = 6$. What is AC?

27. **(2012 AMC 10A #18)** The closed curve in the figure is made up of 9 congruent circular arcs each of length $\frac{2\pi}{3}$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?



28. **(2013 AMC 10A #19)** In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b-representation of 2013 end in the digit 3?

29. **(2013 AMC 12A #15)** Rabbits Peter and Pauline have three offspring—Flopsie, Mopsie, and Cottontail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

30. **(2016 AMC 10A #20)** For some particular value of N, when $(a+b+c+d+1)^N$ is expanded and like terms are combined, the resulting expression contains exactly 1001 terms that include all four variables a, b, c, and d, each to some positive power. What is N?

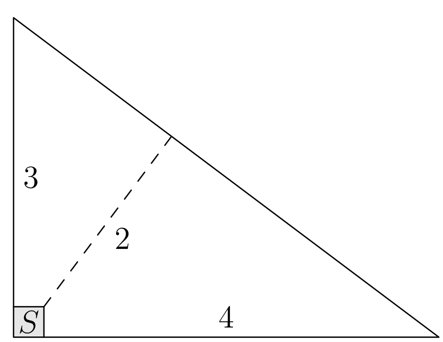
31. **(2008 AMC 12B #16)** A rectangular floor measures a by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half of the area of the entire floor. How many possibilities are there for the ordered pair (a, b)?

32. **(2011 AMC 10B #23)** What is the hundreds digit of 2011^{2011}?

33. **(2012 AMC 10A #23)** Adam, Benin, Chiang, Deshawn, Esther, and Fiona have internet accounts. Some, but not all, of them are internet friends with each other, and none of them has an internet friend outside this group. Each of them has the same number of internet friends. In how many different ways can this happen?

34. **(2013 AMC 10A #23)** In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

35. **(2018 AMC 10A #23)** Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?



36. **(2020 AMC 10A #24)** Let n be the least positive integer greater than 1000 for which

$$\gcd(63, n+120) = 21 \quad \text{and} \quad \gcd(n+63, 120) = 60.$$

What is the sum of the digits of n?

37. **(2020 AMC 12A #21)** How many positive integers n are there such that n is a multiple of 5, and the least common multiple of $5!$ and n equals 5 times the greatest common divisor of $10!$ and n?

38. **(2019 AMC 10A #25)** For how many integers n between 1 and 50, inclusive, is

$$\frac{(n^2-1)!}{(n!)^n}$$

an integer? (Recall that $0! = 1$.)

39. **(2007 AMC 12B #23)** How many non-congruent right triangles with positive integer leg lengths have areas that are numerically equal to 3 times their perimeters?

40. **(2018 AMC 12B #25)** Circles ω_1, ω_2, and ω_3 each have radius 4 and are placed in the plane so that each circle is externally tangent to the other two. Points P_1, P_2, and P_3 lie on ω_1, ω_2, and ω_3 respectively such that $P_1P_2 = P_2P_3 = P_3P_1$ and line P_iP_{i+1} is tangent to ω_i for each $i = 1, 2, 3$, where $P_4 = P_1$. See the figure below. The area of $\triangle P_1P_2P_3$ can be written in the form $\sqrt{a} + \sqrt{b}$ for positive integers a and b. What is $a + b$?