

# Fun and Games

Prepared by Curtis Liu & Sunny Liang for Advanced 1

2024-02-04

## Introduction

This handout contains descriptions of 6 games. Soon, your team of 2 will compete with other teams for points. The team with the most points wins. For the first hour or so, you will have some time to prepare and strategize. Your strategy will often depend on whether you go first or second, so make sure to consider both cases.

When playing, take note of what other teams come up with, we may study some of these strategies in the future. Not all of these games are *solved* (i.e. have a known optimal strategy), but some of them are. Good luck!

## 1 Prisoner's Dilemma

You and your opponent were caught for stealing prime numbers. The integer police don't know which one of you did it, so they offer you both a deal:

- Each team may choose to tell on the other team or stay silent
- If both teams stay silent, both get 1 point
- If both teams tell on the other, both get 0 points
- If one team tells and the other stays silent, the telling team gets 2 points and the silent team loses 1 point.

Simple enough right? But wait, there's more!

We will repeat this process ten times in a row. Your team must write down a set strategy beforehand that may only depend on the other team's guesses. Think of it like writing a computer programming that will play against another team's program on your behalf. Here are a few examples of some (not very good) strategies:

- always stay silent
- if the other team stayed silent on the first turn, tell on all subsequent turns
- do the opposite of whatever the other team did on the previous turn
- stay silent for 3 turns, then do what the opponent did 3 turns ago

Have fun!

## 2 Nim

16 matchsticks are arranged in 4 rows as shown in my awesome drawing below.



Teams take turns taking match sticks away. On each turn, you may remove any number of match sticks from any one row. The team that removes the final matchstick loses.

The winning team wins as many points as the number of matchsticks they removed.

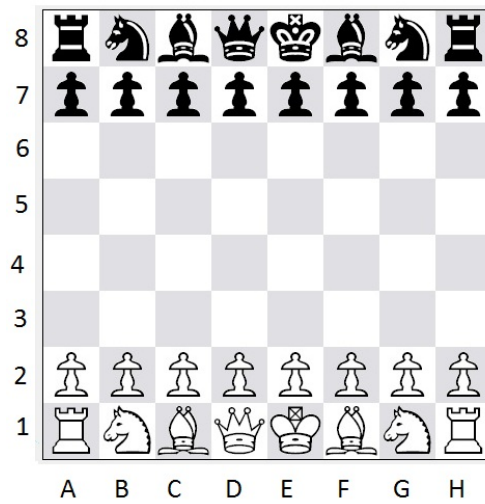
## 3 Dots and boxes

Given 9 dots arranged in a 3-by-3 grid, teams take turns drawing a line between two adjacent dots (no diagonal lines). If, on a turn, you complete a box, you get to mark off the box with an X and win a point. You also get to go again. This goes on until all possible lines are drawn.



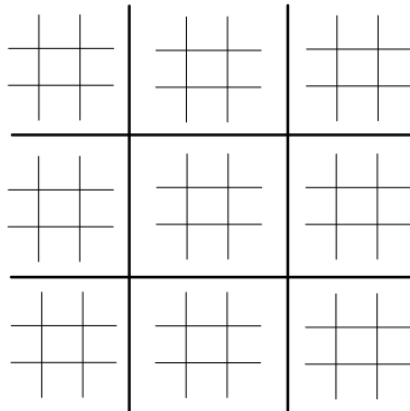
## 4 Jump Chess

Chess... except rooks and queens can jump over other pieces. Winner gets 3 points.



## 5 Ultimate Tic-tac-toe

Tic-tac-toe, except to fill in a square, you have to win a mini-game of tic-tac-toe.



Winner gets 3 points.

## 6 Symbol Strike

Each team starts with 11 spaces on a piece of paper. On the first turn, each team fills in one of the spaces with a symbol that is one of:

- the digits from 0 to 9.
- the four basic arithmetic operators  $+$ ,  $-$ ,  $\times$ , or  $\div$ .

On the next turn, the teams swap papers and add a symbol to one of the other team's spaces. The selection and placement of symbols must follow these rules:

- Operators may not be placed next to each other or at the ends
- No symbol may be used more than once in the same expression

Each team's final expression will be evaluated according to order of operations, and the team with the largest number wins.

Winner gets 5 points.

## Bonus Games

This section will be used if time permits. If it is not covered, try to come up with a strategy at home and challenge your parents to play these games!

### 7 Chips

There are  $n$  chips on the table. Two players, Player One and Player Two, alternate turns by removing chips from the table. On the first move, Player 1 can take any number of chips except for the whole pile and zero. On each following move, the player is not allowed to take more chips than the other player did on the previous move.

For example, if Player 1 made the first move to remove two chips, Player 2 is allowed to remove either one or two chips. If Player 2 chooses to remove one chip, then on the next move, Player 1 is only allowed to remove one chip. The player that takes the last chip is the winner.

### 8 Centipede Game

Alice and Bob are sitting across the table from one another playing the following game. At the beginning, there is \$1 located closer to Alice, on the table. On the first move, Alice can either take the dollar and end the game or she can move the dollar across the table. If she moves the dollar across the table, one more dollar is added to the game.

On his move, Bob can either take \$2 and end the game or he can move the two dollars across the table. In this case, another dollar is added to the game. Alice and Bob alternate taking moves this way. If the game reaches the state with \$100 on the table, it ends and each of the players gets \$50. Find the ways for Alice and Bob to win the largest possible amount of money.