

## INEQUALITIES

MATH CIRCLE (ADVANCED) 10/28/2012

1) Which number is greater

a)  $31^{11}$  or  $17^{14}$ ?

b)  $2^{300}$  or  $3^{200}$ ?

c)  $2^{40}$  or  $3^{28}$ ?

d)  $100^{100}$  or  $150^{50} \cdot 50^{50}$ ?

2) a) Prove that  $2^{100} + 3^{100} < 4^{100}$ .

b) Prove that

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \cdots - \frac{1}{99} + \frac{1}{100} > \frac{1}{5}.$$

3) Prove the following:

a) If  $a \geq b$  and  $x \geq y$ , then  $ax + by \geq ay + bx$ .

b)  $\frac{a^2}{4} + b^2 + c^2 \geq ab - ac + 2bc$  for all  $a, b, c$ .

c) If  $a + b + c = 0$ , then  $ab + bc + ca \leq 0$ .

4) Which number is greater:

a)  $1234567 \cdot 1234569$  or  $1234568^2$ ?

b)  $1234567/7654321$  or  $1234568/7654322$ ?

5) We are given the two fractions

$$\frac{10 \dots 01}{10 \dots 001} \text{ and } \frac{100 \dots 01}{100 \dots 001}$$

where each fraction has one more zero in the denominator than in the numerator. If the numerator in the left fraction has 1984 zeros, and the numerator in the right fraction has 1985 zeros, which of them is greater?

6)\* Prove that

$$\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdots \frac{99}{100} < \frac{1}{10}.$$

7)\* Prove that if  $a_1 \leq a_2 \leq a_3 \leq a_4$  and  $b_1 \leq b_2 \leq b_3 \leq b_4$  then

$$a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4 \geq a_1c_1 + a_2c_2 + a_3c_3 + a_4c_4,$$

where  $c_1, c_2, c_3, c_4$  is an arbitrary permutation of the numbers  $b_1, b_2, b_3, b_4$ .

Hint: You can get any permutation by successively switching two numbers at a time (called a transposition).

Some problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”