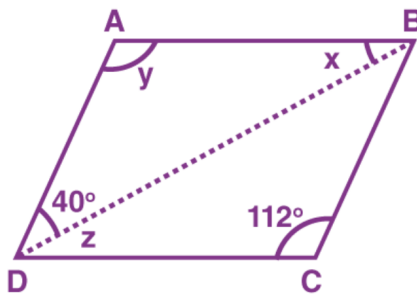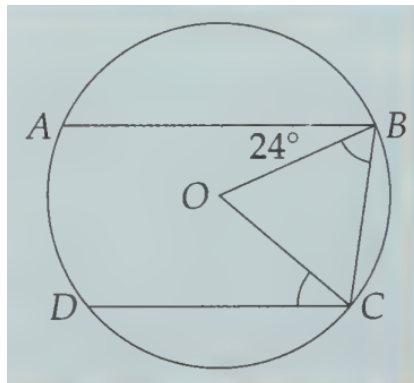# Area and Perimeter

1   Review

Problem 1.1. *Name 3 different congruence tests for triangles.*

Problem 1.2. *Let $\triangle ABC$ and $\triangle DEF$ be two triangles. Suppose we have $AC = DF$ and $\angle ABC = \angle DEF$. Write down one more condition we need to prove $\triangle ABC \simeq \triangle DEF$.*

Problem 1.3. *In the figure below, $ABCD$ is a parallelogram. Find x, y, z.*

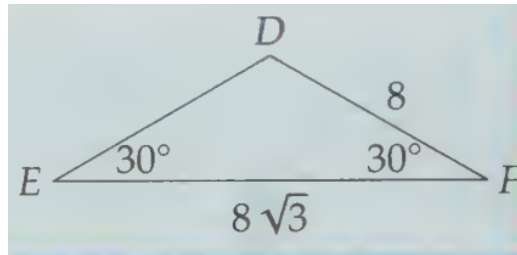**Problem 1.4.** *In the figure below, O is the center of the circle, AB||DC. Find ∠BOC.*

2   Perimeter

Definition 1. *The **perimeter** of a closed figure is how far you travel if you walk along its boundary all the way around it once.*

Problem 2.1. *△ABC is an equilateral triangle with Perimeter 36. Find the side length AB.*

Problem 2.2. *Find the perimeter of the triangle in the diagram below:*

Problem 2.3. *(1) Prove that congruent triangles must have the same perimeter.*

(2) Must two triangles with the same perimeter be congruent? Why or why not?

(3) Must two triangles with the same perimeter be congruent if the triangles have one angle measure in common? Prove it or give an counterexample.

3 Area

Area is a number that measures the total space taken up by a flat (2-D) surface or shape of an object. Roughly speaking, we measure the area of a shape by counting how many 1×1 squares must be used to cover the shape.

Definition 2. *The area of a rectangle with side lengths a and b is defined to be $a \times b$. For any other figure, the area is the minimal total area of rectangles needed to cover the figure entirely.*

Problem 3.1. *Let $ABCD$ be a rectangle with $AB = 4$ and $BC = 5$. Find the area of $ABCD$.*

Problem 3.2. *What is the area of a square $ABCD$ if the perimeter of the square is 36?*

Problem 3.3. *The length of one side of a rectangle is 4 less than 3 times that of an adjacent side. The perimeter of the rectangle is 64. Find the area of the rectangle.*

Problem 3.4. *The perimeter of a square garden is 64 meters. The path surrounding the garden has uniform width and has an area of 228 square meters. How many meters of fencing are needed to surround the outer edge of the path?*
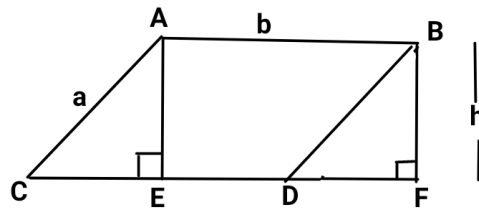

Problem 3.5. *Let $\triangle ABC$ be a right triangle with $\angle A = 90°$. Suppose $AB = 4$ and $BC = 5$, find the area of $\triangle ABC$. Prove your result using Problem 3.1.*

Deduce a formula for computing areas of right triangles.

Problem 3.6. *In the figure below, $\angle ZXY = \angle XWZ = 90°$, $XZ = 6$, $XY = 8$, $ZY = 10$. Find the length XW.*
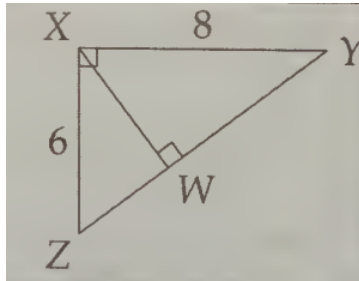
Problem 3.7. *(1) Suppose $ACDB$ is a parallelogram. prove that $\triangle ACE \simeq \triangle BDF$.*


(2) Explain how the area of $ACDB$ is the same as the area of the rectangle $AEFB$. Dudece an area formula for any parallelogram.

Problem 3.8. *The perimeter of a parallelogram $ABCD$ is 100. What shape does $ABCD$ need to be in order for the area to be maximized? Explain your answer using the area formula.*

Problem 3.9. *Let $\triangle ABC$ be any arbitrary triangle. Find, with proof, a formula for area in the following two ways:*

(1) Let h be the height from B to the straight line AC, as shown below. Find the areas of $\triangle BCE$ and $\triangle BEA$, then add them up.
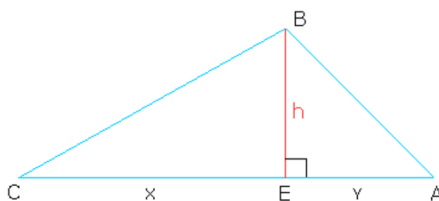
(2) Place $\triangle ABC$ into a parallelogram $ABCD$ as below. Explain why the area of $\triangle ABC$ is half of the area of $ABCD$.
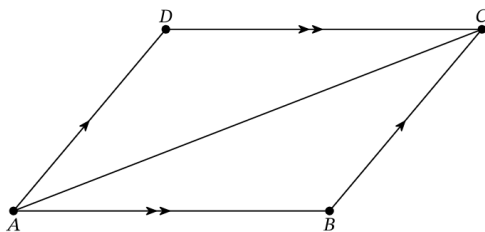
Problem 3.10. *Let E and F be the midpoints of the sides BC and AD of parallelogram ABCD. Find the area of the quadrilateral formed by the lines AE, ED, BF and FC if it is known that the area of ABCD is S.*

Problem 3.11. *The diagonals of quadrilateral ABCD meet at point O. Prove that the area of $\triangle AOB$ is equal to that of $\triangle COD$ if and only if $BC \| AD$.*

Problem 3.12. *(Challenge) The height of a trapezoid whose diagonals are mutually perpendicular is equal to 4. Find the area of the trapezoid if it is known that the length of one of its diagonals is equal to 5.*

Problem 3.13. *(Challenge)* *In the diagram below, ABCD is a quadrilateral (but not a parallelogram). X is the intersection of the diagonals AC and BD. The area of ABX is 24, the area of DCX is 10, and the area of BXC is 15. Find the area of ADX.*
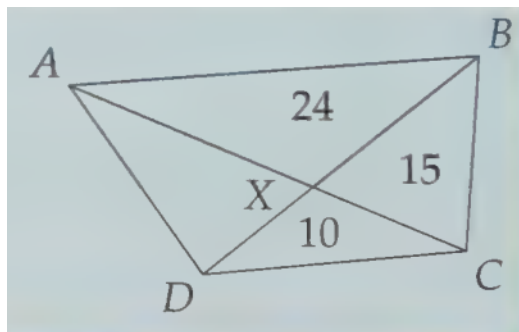
Problem 3.14. *(Challenge)* *Let ABCD be a trapezoid with AB||CD, AB = 11, BC = 5, CD = 19, and DA = 7. Bisectors of ∠A and ∠D meet at P, and bisectors of ∠B and ∠C meet at Q. What is the area of hexagon ABQCDP?*