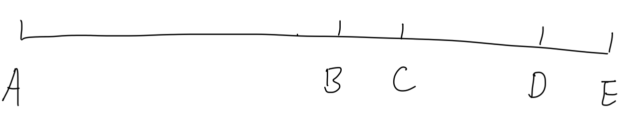
Euclidean Geometry

Sunday, January 7, 2024 8:02 AM

1.2

$AB = 16$

1.3

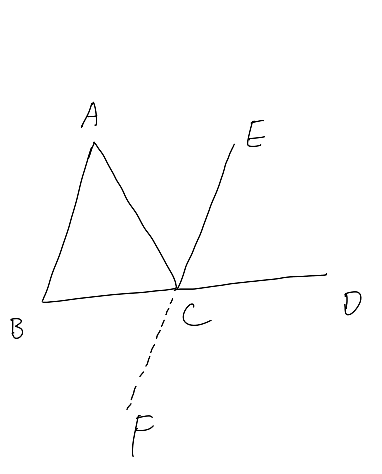


$AD = \frac{7}{8} AE$

1.4 17

1.6 If $\angle A + \angle B \neq 180^\circ$, then L_1 and L_2 are not parallel

1.7



① Since $AB \parallel CE$, $\angle BAC + \angle ACF = 180^\circ$

But since $\angle ACF + \angle ACE = 180^\circ$

$\angle ACF + \angle ACE = \angle BAC + \angle ACF$

So $\angle ACE = \angle BAC$

② Since $AB \parallel CE$, $\angle ABC + \angle BCE = 180^\circ$

Since $\angle BCE + \angle ECD = 180^\circ$,

$\angle ABC + \angle BCE = \angle BCE + \angle ECD$

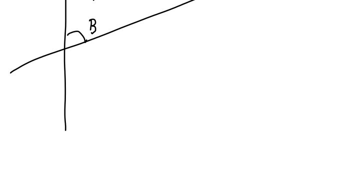
So $\angle ABC = \angle ECD$

1.9

$30^\circ, 60^\circ, 90^\circ$

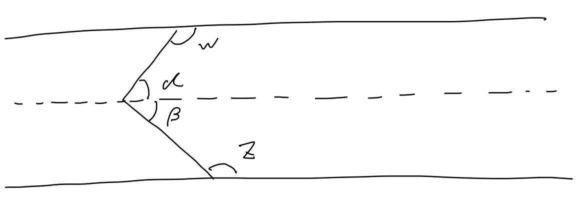
1.10

If the two lines are not parallel, they intersect and thus form a triangle



We have $\angle A + \angle B = 180^\circ - \angle C < 180^\circ$

1.11



$\alpha = 180^\circ - 135^\circ = 45^\circ$

$\beta = 180^\circ - 147^\circ = 33^\circ$

$x = \alpha + \beta = 78^\circ$

1.12 Let $\angle A''AB = x$, $\angle BCC'' = y$

$2x + 2y = 180^\circ$

$\angle ABC = x + y = 90^\circ$

1.13 90°

2.2

1, 2, 4

2.3 $\frac{AC}{AB} = \frac{AB}{AD} \Rightarrow AB^2 = 4 \times 9$

$AB = 6$

2.4 $\triangle APQ \sim \triangle ABC$

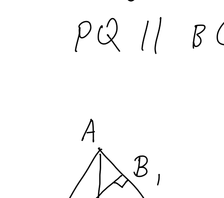
$\Downarrow$

$\angle APQ = \angle ABC$

$\Downarrow$

$PQ \parallel BC$

2.5



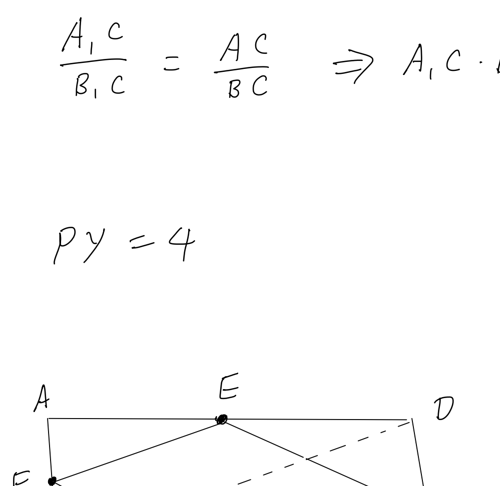
$\triangle AA_1C \sim \triangle BB_1C$

$\frac{A_1C}{B_1C} = \frac{AC}{BC} \Rightarrow A_1C \cdot BC = B_1C \cdot AC$

2.6

$PY = 4$

2.7



By problem 2.4, $BD \parallel EF$, $BD \parallel HG$

So $EF \parallel HG$

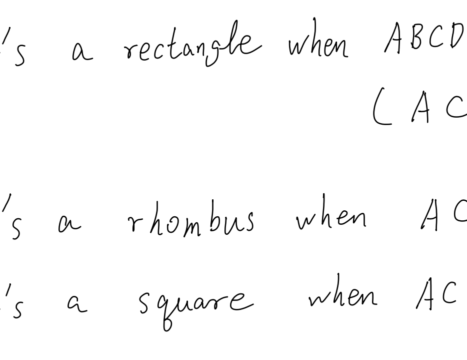
Similarly, $FG \parallel EH$

It's a rectangle when ABCD is a kite ($AC \perp BD$)

It's a rhombus when $AC = BD$

It's a square when $AC = BD$, $AC \perp BD$

2.8



$\triangle ABK \sim \triangle MDK$

$\frac{AK}{KM} = \frac{BK}{KD}$

$\triangle BKL \sim \triangle DKA$

$\frac{BK}{KD} = \frac{KL}{AK}$

So $\frac{AK}{KM} = \frac{KL}{AK}$, $AK^2 = KM \cdot LK$

2.9

$\frac{FM}{FE} = \frac{FQ}{FP} = \frac{LO}{OC} = \frac{1}{3}$

(Since LC and AK are medians)

Similarly,

$\frac{NE}{FE} = \frac{1}{3}$