Intermediate 1 **Jan 14, 2024**

Angles

1 Postulates and Definitions Review

Last time, we introduced geometric objects through Euclid's postulates, which are as follows:

Postulate 1: *A straight line may be drawn from any one point to any other point.*

Postulate 2: *A line segment can be produced indefinitely in a straight line.*

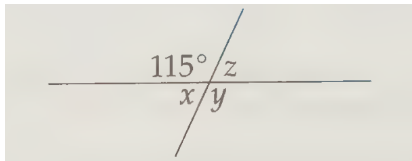
Postulate 3: *A circle can be drawn with any given center and radius.*

Postulate 4: *All right angles are equal. (By convention, we define* 1° *to be 1/90 of a right angle, so a right angle is* 90°*.)*

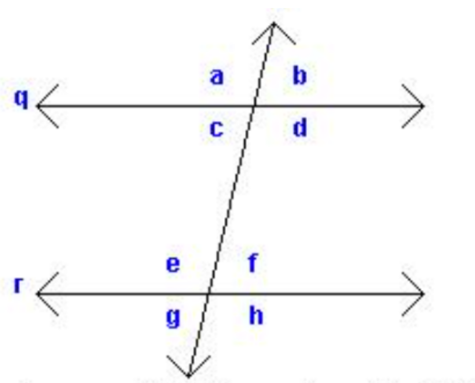
Postulate 5: *If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.*

Problem 1.1. *How many right angles are there in a complete angle (an angle of* 360°*)?*

Problem 1.2. *Find x, y, z in the diagram*



Definition 1. *Suppose one straight line cuts across two other straight lines:*



- *The angles a and e are called* ***corresponding angles***.
- *The angles c and f are called* ***alternate interior angles***.
- *The angles a and h are called* ***alternate exterior angles***.
- *The angles c and e are called* ***same-side interior angles***.
- *The angles a and g are called* ***same-side exterior angles***.

Problem 1.3. *In the previous diagram, find all pairs of corresponding angles.*

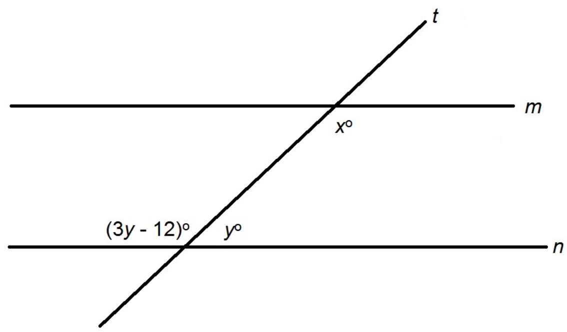
Recall from last time that we have proven parts of the following theorem:

Theorem 1. *If a straight line l cuts across two other straight lines q and r, then the following are equivalent:*

- $q||r$.
- *Corresponding angles are equal.*
- *Alternate interior angles are equal.*
- *Alternate exterior angles are equal.*
- *Same-side interior angles sum up to* $180°$.
- *Same-side exterior angles sum up to* $180°$.

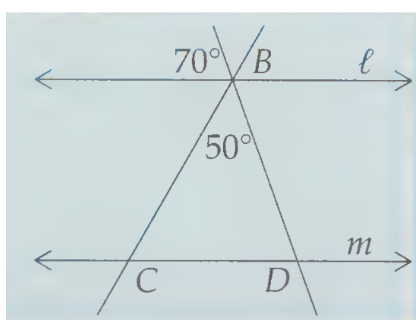
Problem 1.4. *In the previous diagram, $q||r$. We know that the angle $\angle a = 135°$. Find the measure of b, c, d, e, f, g, h.*

Problem 1.5. *In the diagram below, the straight lines $m||n$. We also measure three angles as shown below in terms of x and y. Find x and y.*

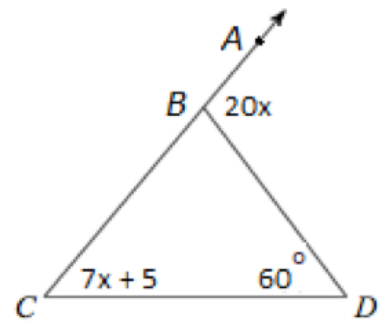


2 Angles and Triangles

Problem 2.1. *In the diagram below, we know that $l||m$. Find $\angle BDC$ and $\angle BCD$.*

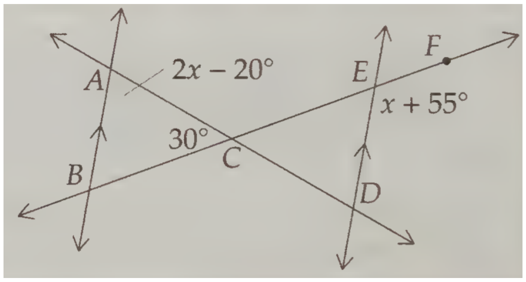


Problem 2.2. *Find x.*

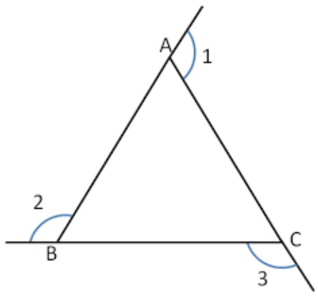


Problem 2.3. *In the previous problem, the angle x is called an **exterior angle** of the triangle ΔABC. Using the fact that the sum of interior angles in a triangle is always $180°$, show that an exterior angle always equal to the sum of the other two interior angles in a triangle.*

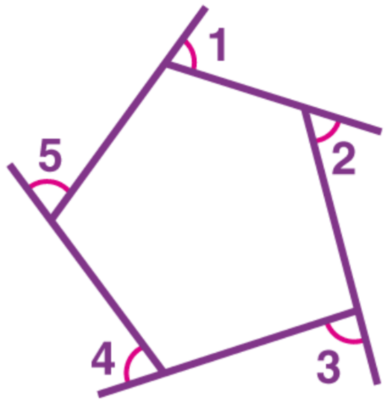
Problem 2.4. *In the following diagram, $AB || ED$. We know that $\angle ACB = 30°$, $\angle BAC = 2x - 20°$, and $\angle DEF = x + 55°$. Find x and $\angle CED$.*



Problem 2.5. *What is $\angle 1 + \angle 2 + \angle 3$? Explain why your result is true using problem 2.3.*



Problem 2.6. *What is* $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5$*? Prove it.*



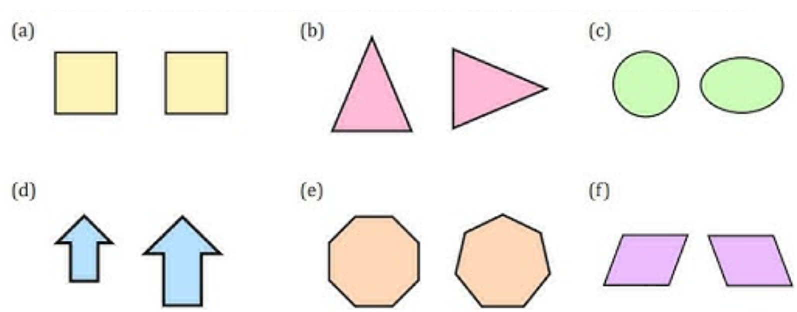
Problem 2.7. *Is it possible for the exterior angles of a triangle to be in the ratio* $1 : 2 : 6$*? Sketch such triangle or explain why it does not exist.*

3 Congruent Triangles

Definition 2. *Two geometric figures are* ***congruent*** *if they are exactly the same after some combination of translation, rotation, and flipping.*

In other words, two polygons are congruent if they have the same number of sides, the corresponding sides each have the same length, and the corresponding angles each have the same degree.

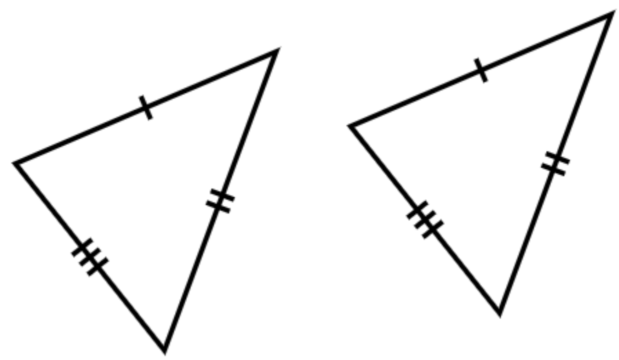
Problem 3.1. *Which of the following pairs of figures are congruent?*



Problem 3.2. *State a necessary and sufficient condition for two circles being congruent.*

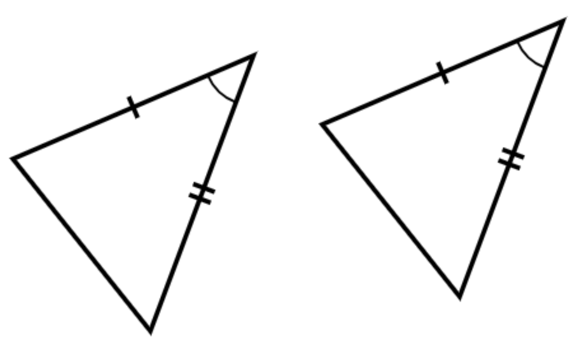
Two triangles are congruent if they have exactly the same side lengths and angles. However, checking every single side length and angle can be burdensome. In place, mathematicians have found multiple simpler conditions to determine whether two triangles are congruent. These conditions are known as the congruence tests.

SSS Congruence: Two triangles are congruent if the lengths of each side of one triangle are equal to the lengths of the corresponding sides of the other triangle.



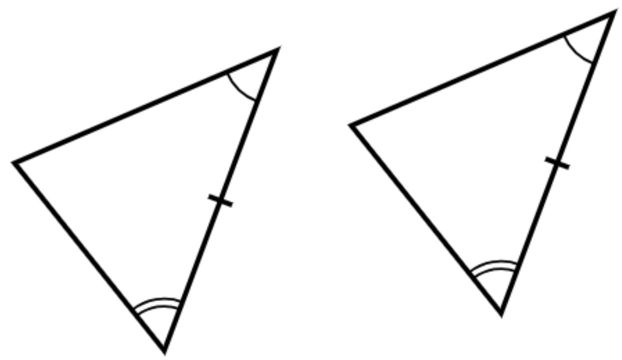
This is called side-side-side congruence, or simply SSS.

SAS Congruence: Two triangles are congruent if two pairs of corresponding sides and the corresponding angles between them are equal.



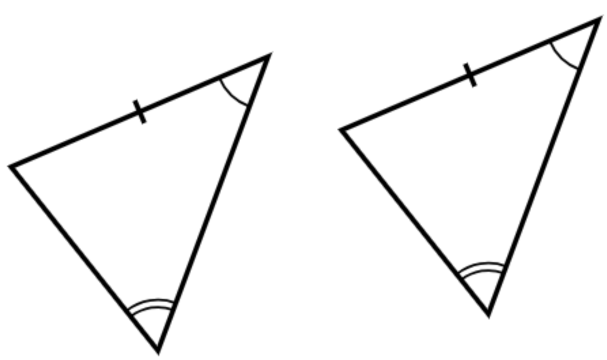
This is called side-angle-side congruence, or simply SAS.

ASA Congruence: Two triangles are congruent if two pairs of corresponding angles and the corresponding sides between them are equal.



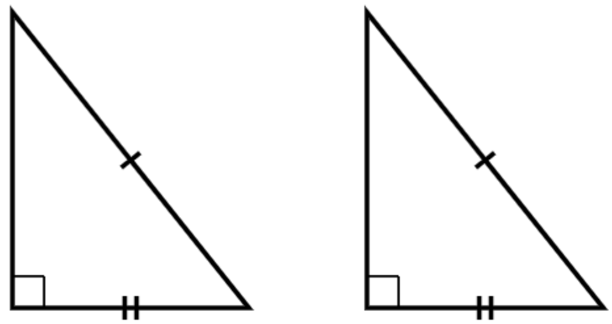
This is called angle-side-angle congruence, or simple ASA.

AAS Congruence: Two triangles are congruent if two pairs of corresponding angles and one pair of corresponding sides (not between the angles) are equal.



This is called angle-angle-side congruence, or simple AAS.

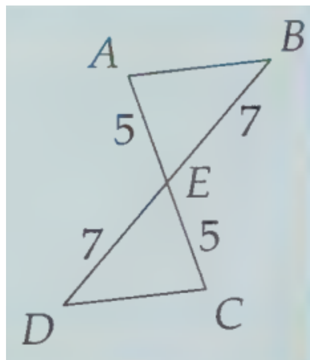
HL Congruence: Two <u>right</u> triangles (triangles with a right angle) are congruent if the pair of hypotenuses and another pair of corresponding sides are equal.



Note 1. *We use the notation $\triangle ABC \simeq \triangle A'B'C'$ to denote that the triangle ABC is congruent to triangle $A'B'C'$. Moreover, the side AB corresponds to $A'B'$, the side BC corresponds to $B'C'$, the side CA corresponds to $C'A'$.*

Problem 3.3. *Why is there no congruence test named SSA? Draw two triangles such that two pairs of corresponding sides and a pair of corresponding angles are equal, but the two triangles are not congruence.*

Problem 3.4. *In the diagram below, point E is the midpoint of both $\overline{AC}$ and $\overline{BD}$. Which congruence test would you use to show that $\triangle ABE \simeq \triangle CDE$?*

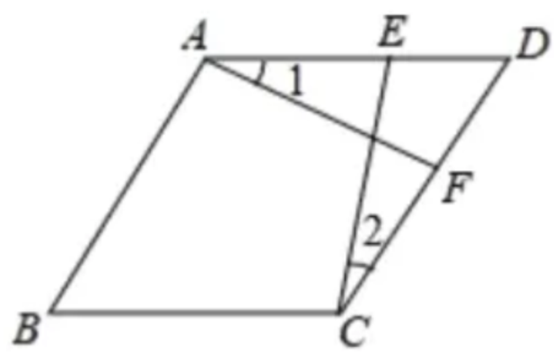


Explain why $AB || CD$.

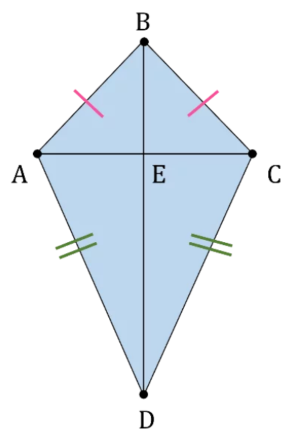
Problem 3.5. *In $\triangle ABC$, $AB = AC$. Suppose AD is the height of $\triangle ABC$, explain how $\triangle ABD \simeq \triangle ACD$ using HL congruence test.*

Problem 3.6. *State a set of conditions such that two quadrilaterals are congruent. Try to use the least number of conditions but make sure that the conditions guarantee the congruence. (Hint: a quadrilateral can be divided into two triangles.)*

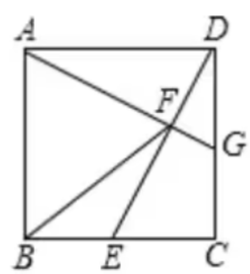
Problem 3.7. *In the rhombus $ABCD$, E, F are points on the sides $\overline{AD}, \overline{CD}$. Suppose $AE = CF$. Prove that $\angle 1 = \angle 2$.*



Problem 3.8. ***(Challenge)*** *In the diagram below, $AB = BC$ and $AD = CD$. Show that BD is perpendicular to AC.*



Problem 3.9. ***(Challenge)*** *In the figure below, $ABCD$ is a square, E is the midpoint of $\overline{BC}$. The line segment $\overline{AG}$ is perpendicular to DE. Show that $\triangle ADG \simeq \triangle DCE$.*



Problem 3.10. ***(Challenge)*** *In $\triangle ABC$ below, $\angle BAC = 90°$, $\overline{AB} = \overline{AC}$. Suppose AD is the height of $\triangle ABC$, the points E, F are on the sides $\overline{AB}$ and $\overline{AC}$ such that DE is perpendicular to DF. Prove that $\overline{AF} = \overline{BE}$.*