Review Game: Number Theory

December 10, 2023

Luka is making lemonade to sell at a school fundraiser. His recipe requires 4 times as much water as sugar and twice as much sugar as lemon juice. He uses 3 cups of lemon juice. How many cups of water does he need? **(1 point)**

(A) 6   (B) 8   (C) 12   (D) 18   (E) 24

How many integers between 2020 and 2400 have four distinct digits arranged in increasing order? (For example, 2347 is one integer.) **(1 point)**

(A) 9   (B) 10   (C) 15   (D) 21   (E) 28

How many factors of 2020 have more than 3 factors? (As an example, 12 has 6 factors, namely 1,2,3,4,6, and 12. **(1 point)**

(A) 6   (B) 7   (C) 8   (D) 9   (E) 10

What is the value of the product $\left(\frac{1\cdot3}{2\cdot2}\right)\left(\frac{2\cdot4}{3\cdot3}\right)\left(\frac{3\cdot5}{4\cdot4}\right)\cdots\left(\frac{97\cdot99}{98\cdot98}\right)\left(\frac{98\cdot100}{99\cdot99}\right)$? **(2 points)**

$(A)1\frac{1}{2}$ (B) $\frac{50}{99}$ (C) $\frac{9800}{9801}$ (D) $\frac{100}{99}$ (E) 50

In a tournament there are six teams that play each other twice. A team earns 3 points for a win, 1 point for a draw, and 0 points for a loss. After all the games have been played it turns out that the top three teams earned the same number of total points. What is the greatest possible number of total points for each of the top three teams? **(2 points)**

(A) 22   (B) 23   (C) 24   (D) 26   (E) 30

A store increased the original price of a shirt by a certain percent and then decreased the new price by the same amount. Given that the resulting price was 84% of the original price, by what percent was the price increased and decreased? **(3 points)**

(A) 16   (B) 20   (C) 28   (D) 36   (E) 40

After Euclid High School's last basketball game, it was determined that $\frac{1}{4}$ of the team's points were scored by Alexa and $\frac{2}{7}$ were scored by Brittany. Chelsea scored 15 points. None of the other 7 team members scored more than 2 points What was the total number of points scored by the other 7 team members? **(3 points)**

(A) 10   (B) 11   (C) 12   (D) 13   (E) 14