Review Game: Algebra

December 10, 2023

Suppose \( n^* \) means \( \frac{1}{n} \), the reciprocal of \( n \). For example, \( 5^* = \frac{1}{5} \). How many of the following statements are true? (1 point)

(i) \( 3^* + 6^* = 9^* \)
(ii) \( 6^* \cdot 4^* = 2^* \)
(iii) \( 2^* \cdot 6^* = 12^* \)
(iv) \( 10^* \div 2^* = 5^* \)

(A) 0  (B) 1  (C) 2  (D) 3  (E) 4

Pick two consecutive positive integers whose sum is less than 100. Square both of those integers and then find the difference of the squares. Which of the following could be the difference? (1 point)

(A) 2  (B) 64  (C) 79  (D) 96  (E) 131

A multiple choice examination consists of 20 questions. The scoring is +5 for each correct answer, 2 for each incorrect answer, and 0 for each unanswered question. John’s score on the examination is 48. What is the maximum number of questions he could have answered correctly? (1 point)

(A) 9  (B) 10  (C) 11  (D) 12  (E) 13

The product of the two 99-digit numbers 303, 030, 303, ..., 030, 303 and 505, 050, 505, ..., 050, 505 has thousands digit \( A \) and units digit \( B \). What is the sum of \( A \) and \( B \)? (2 point)

(A) 3  (B) 5  (C) 6  (D) 8  (E) 10

A fifth number, \( n \), is added to the set of numbers 3, 6, 9, 10 to make the mean of the set of five numbers equal to its median. The number of possible values for \( n \) is: (2 point)

(A) 1  (B) 2  (C) 3  (D) 4  (E) more than 4
Which of the following sets of whole numbers has the largest average? (3 point)

(A) multiples of 2 between 1 and 101
(B) multiples of 3 between 1 and 101
(C) multiples of 4 between 1 and 101
(D) multiples of 5 between 1 and 101
(E) multiples of 6 between 1 and 101

The polynomial \( p(x) = x^3 + ax^2 + bx + c \) has the property that the average of its zeros, the product of its zeros, and the sum of its coefficients are all equal. The \( y \)-intercept of the graph of \( y = p(x) \) is 2. What is \( b \)? (3 point)

(A) -11  (B) -10  (C) -9  (D) 1  (E) 5