OLGA RADKO MATH CIRCLE: ADVANCED 3

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Fall Final Exam II

Name: _____

Problem 1	/10
Problem 2	/10
Problem 3	/10
Problem 4	/10
Problem 5	/10
Total	/50

Problem 1:

How many solutions does the equation $34x = 51$ in $\mathbb{Z}/170\mathbb{Z}$?

How many zeroes does the polynomial $x^3 + 3$ have in $\mathbb{Z}/6\mathbb{Z}$?

Solution 1:

Problem 2:

(1) Let R be a ring such that for any element x in R, we have that $x^2 = x$. Prove that the characteristic of R is 2.
(2) Let S be a ring that is not a field, such that for any non-invertible element x in S, we have that $x^2 = x$. Prove that the characteristic of S is 2.

Solution 2:

We say that two rings R and S are *isomorphic* if there exists a function $\varphi : R \to S$, such that:

- φ is a bijective function.
- $\varphi(a+b) = \varphi(a) + \varphi(b)$.
- $\varphi(ab) = \varphi(a)\varphi(b)$
- $\varphi(1_R) = 1_S$

We say that φ is an isomorphism.

Problem 3:

(1) Show that an isomorphism sends zero-divisors to zero-divisors
(2) Show that an isomorphism sends invertible elements to invertible elements
(3) Show that if a ring R is isomorphic to a field F, then R is a field.

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Solution 3:

Problem 4: Show that no two of $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ are isomorphic.
Solution 4:

Problem 5: Show that $x^{10} + x^3 + 1$ is irreducible in $\mathbb{F}_2[x]$.

You may use the following fact: "*Any irreducible polynomial* $p(x)$ *of degree* d *in* $\mathbb{F}_p[x]$ *divides the polynomial* $x^{p^d-1} - 1$"

Solution 5:

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