OLGA RADKO MATH CIRCLE: ADVANCED 3

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Fall Final Exam I

Name: 

| Problem 1 | /10 |
| Problem 2 | /10 |
| Problem 3 | /10 |
| Problem 4 | /10 |
| Problem 5 | /10 |
| Total     | /50 |
Say if the following statements are True or False. Prove the True ones and give a counterexample for the False ones

**Problem 1:**

___ The intersection of two ideals is an ideal.

___ The intersection of two subrings of a ring $R$ is a subring of $R$.

**Solution 1:**
Problem 2:

___ The set of all non-invertible elements in a field forms an ideal

___ The set of all non-invertible elements in a ring forms an ideal.

Solution 2:
Problem 3:

__ A ring $R$ is a field if and only if its only ideals are $\{0\}$ and $R$.

__ A ring $R$ is a field if and only if it contains no subring other than itself.

Solution 3:
Problem 4:

___ If $F$ is a field and $R$ is a subring of $F$, then $R$ is a field.

___ If $R$ is a ring that is not a field, and $S$ is a subring of $R$. Then $S$ is not a field.

Solution 4:
Problem 5:

___ If $R$ and $S$ are rings containing no zero-divisors, then $R \times S$ contains no zero-divisor.

___ If $R$ is a finite ring containing no zero divisors, then $R$ is a field.

Solution 5: