More Sets

Sunday, November 19, 2023 6:10 PM

1.1.

$A \cup B = \{1, 2, 3, 4, 6, 8\}$

$A \cap B = \{2, 4\}$

$A \setminus B = \{1, 3\}$

$B \setminus A = \{6, 8\}$

1.2

$\left\lfloor \frac{100}{7} \right\rfloor = 14$

1.3

(a) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$|A \cup B \cup C| = 9$

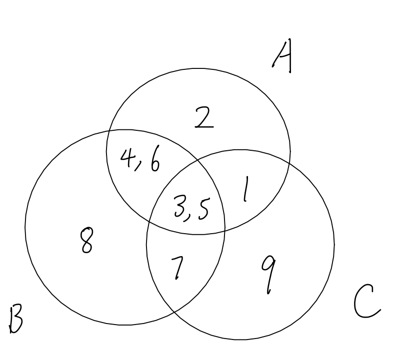
(b) $|A \cap B| = |\{3, 4, 5, 6\}| = 4$

$|B \cap C| = |\{3, 5, 7\}| = 3$

$|A \cap C| = |\{1, 3, 5\}| = 3$

$|A \cap B \cap C| = |\{3, 5\}| = 2$

(c)



(d) $|A \cup B \cup C| = 9$

$|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= 6 + 6 + 5 - 4 - 3 - 3 + 2$

$= 9$

1.4

$\mathbb{Q} \setminus \mathbb{Z} = \left\{ \frac{a}{b} : a \in \mathbb{Z},\ b \in \mathbb{N} \setminus \{0, 1\},\ \gcd(a, b) = 1 \right\}$

1.5

$\{a \in \mathbb{Z} : a \text{ is even}\}$

1.6

$\emptyset$ or Empty set or $\{\}$

2.1. Yes since they are subsets of A.

2.2

$\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$

2.3

$\{\emptyset\}$

Warning: $\emptyset$ is incorrect.

2.4

$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n$

2.5

$P(A) = \{\emptyset, \{0\}, \{1\}, \{0,1\}\}$

$$P(P(A)) = \left\{ \begin{array}{l} \emptyset, \{\emptyset\}, \{\{0\}\}, \{\{1\}\}, \{\{0,1\}\}, \\ \{\emptyset, \{0\}\}, \{\emptyset, \{1\}\}, \{\emptyset, \{0,1\}\}, \\ \{\{0\}, \{1\}\}, \{\{0\}, \{0,1\}\}, \\ \{\{1\}, \{0,1\}\}, \{\emptyset, \{0\}, \{1\}\} \\ \{\emptyset, \{0\}, \{0,1\}\}, \{\emptyset, \{1\}, \{0,1\}\} \\ \{\{0\}, \{1\}, \{0,1\}\}, \{\emptyset, \{0\}, \{1\}, \{0,1\}\} \end{array} \right\}$$

3.1

$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

3.2

$|A \times B| = 6 = |A| \cdot |B|$

3.3.

$A \times B = \{(a, b) : a \in A,\ b \in B\}$

For each (a, b), there are $|A|$ choices for a, $|B|$ choices for b, so $|A| \times |B|$ choices for (a, b) in total.

3.4. $\mathbb{Z} \times \mathbb{Q}$ is the set of pairs (a, b) where a is an integer, and b is a rational number.

3.5 & 3.6

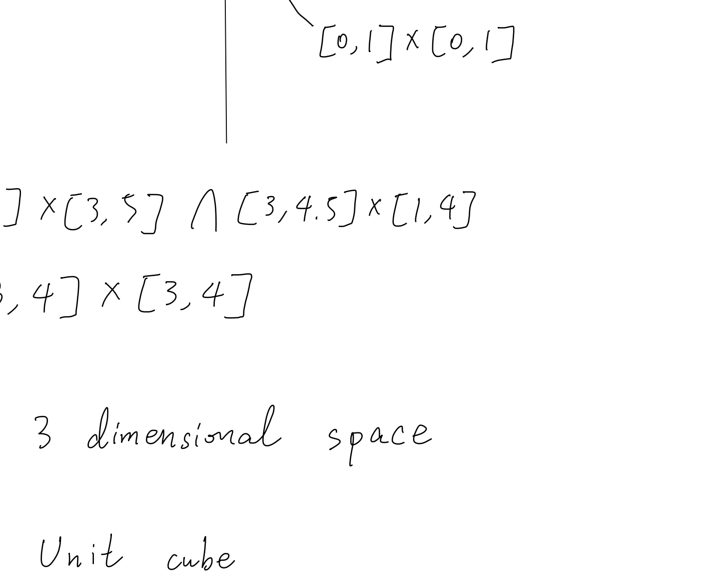
3.7

3

3.8

3.9 $(5, 30)$

3.10 & 3.11



$[2, 4] \times [3, 5] \cap [3, 4.5] \times [1, 4]$

$= [3, 4] \times [3, 4]$

3.12 3 dimensional space

3.13 Unit cube

3.14 Unit disk

3.15 Cylinder with radius 1, height 1

3.16 Doughnut / Solid torus