Sets

Monday, November 6, 2023 4:00 AM

1.6. $0.55 = \frac{55}{100} = \frac{11}{20}$

1.12 Each $a \in A$ can be included or excluded from a subset, so the total numbers of ways to form a subset is 2^n.

Or

By induction: If $n=0$, $A = \emptyset$

If $n > 0$, pick $a \in A$, then

$$\begin{aligned}\#\text{ of subsets of } A &= \#\text{ of subsets including } a \\ &\quad + \#\text{ of subsets excluding } a \\ &= 2^{n-1} + 2^{n-1} \\ &= 2^n\end{aligned}$$

1.13 $\begin{cases} 0 & \text{if } A = \emptyset \\ 2^n - 2 & \text{otherwise} \end{cases}$

2.3 $A \cup B$ is defined as the set of x such that x is in A and x is in B.

2.10 $B \setminus A := \{x : x \in B \text{ and } x \notin A\}$

2.11 $100 - \left\lfloor \frac{100}{3} \right\rfloor = 100 - 33 = 67$

($\lfloor x \rfloor$ means the largest integer less than or equal to x)

2.15 Salt: $17 + 25 = 42$

Salt only: 17

Pepper: $42 + 25 = 67$

Pepper only: 42

Salt and peper: 25

Neither: $100 - 42 - 25 - 17 = 16$

2.16

Trains: $21 + 9 + 8 + 10 = 48$

Planes: $17 + 9 + 8 + 6 = 40$

Cars: $12 + 10 + 8 + 6 = 36$

Trains and Planes: $9 + 8 = 17$

Trains, planes, not cars: 9

Trains or cars, not planes: $21 + 10 + 12 = 43$

All: $21 + 12 + 17 + 9 + 10 + 6 + 8 = 83$

None: $100 - 83 = 17$

2.17

$|A| = |A \setminus B| + |A \cap B|$

$|B| = |B \setminus A| + |A \cap B|$

$|A \cup B| = |A \setminus B| + |A \cap B| + |B \setminus A|$

So $|A| + |B| - |A \cap B|$

$= |A \setminus B| + |B \setminus A| + 2|A \cap B| - |A \cap B|$

$= |A \cup B|$

2.18. Love either one: $25 + 22 - 16 = 31$

neither: $40 - 31 = 9$

2.19

1 If $x \in (A \cup B) \cap C$, then

$x \in A \cup B$ and $x \in C$

So $x \in A$ and $x \in C$, or $x \in B$ and $x \in C$

So $x \in (A \cap C) \cup (B \cap C)$.

Conversely, if $x \in (A \cap C) \cup (B \cap C)$, then

$x \in A \cap C$ or $x \in B \cap C$

In either case, $x \in C$, and either $x \in A$ or $x \in B$. So $x \in (A \cup B) \cap C$.

This shows that $(A \cup B) \cap C$ and $(A \cap C) \cup (B \cap C)$ contains the exact same elements, so they are equal.

2 Let $x \in (A \cap C) \cup B$, then

$x \in A \cap C$ or $x \in B$

If $x \in A \cap C$, then x is in both A and C, so x must be in both $A \cup B$ and $B \cup C$, so

$x \in (A \cup B) \cap (B \cup C)$

If $x \in B$, then $x \in A \cup B$ and $x \in B \cup C$, so $x \in (A \cup B) \cap (B \cup C)$

Conversely, let $x \in (A \cup B) \cap (B \cup C)$, then x is in either A or B, and x is either in B or C. If x is not in B, then x must be in A and in C, so

$x \in (A \cap C) \cup B$

If $x \in B$, then $x \in (A \cap C) \cup B$ again.

So $(A \cap C) \cup B$ and $(A \cup B) \cap (B \cup C)$ have the exact same elements.

2.20.

$|A| = |A \text{ only}| + |(A \cap B) \setminus C| + |(A \cap C) \setminus B| + |A \cap B \cap C|$

$|B| = |B \text{ only}| + |(A \cap B) \setminus C| + |(B \cap C) \setminus A| + |A \cap B \cap C|$

$|C| = |C \text{ only}| + |(A \cap C) \setminus B| + |(B \cap C) \setminus A| + |A \cap B \cap C|$

$|A \cap B| = |(A \cap B) \setminus C| + |A \cap B \cap C|$

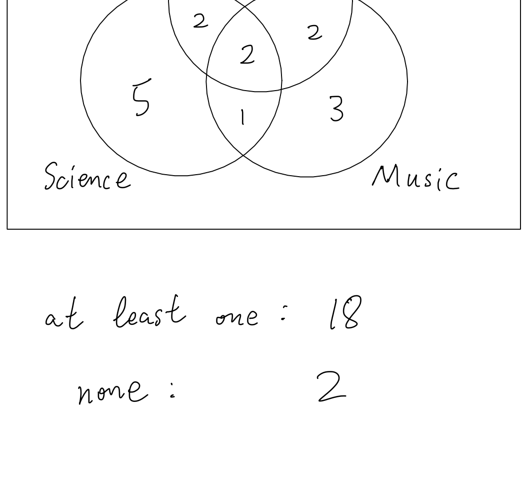
$|B \cap C| = |(B \cap C) \setminus A| + |A \cap B \cap C|$

$|A \cap C| = |(A \cap C) \setminus B| + |A \cap B \cap C|$

$$\begin{aligned}|A \cup B \cup C| &= |A \text{ only}| + |B \text{ only}| + |C \text{ only}| \\ &\quad + |(A \cap B) \setminus C| + |(B \cap C) \setminus A| + |(A \cap C) \setminus B| \\ &\quad + |A \cap B \cap C|\end{aligned}$$

Using these equations to expand both sides of the equation we want to prove.

2.21



at least one: 18

none: 2

2.22

$\left\lfloor \frac{100}{2} \right\rfloor + \left\lfloor \frac{100}{3} \right\rfloor + \left\lfloor \frac{100}{5} \right\rfloor - \left\lfloor \frac{100}{6} \right\rfloor - \left\lfloor \frac{100}{10} \right\rfloor - \left\lfloor \frac{100}{15} \right\rfloor + \left\lfloor \frac{100}{30} \right\rfloor$

$= 50 + 33 + 20 - 16 - 10 - 6 + 3$

$= 74$

$100 - 74 = \boxed{26}$