Intermediate 1 **Oct 29, 2023**

Think Outside the Box!

1. Peter exchanges stickers with his friends. For every sticker he gives someone, he gets 5 stickers back. Suppose he starts the exchange with just one sticker. How many stickers will he have after 30 exchanges?

   Each exchange adds 4 stickers, so the total number is

   $$1 + 4 \times 30 = 121.$$

   .

2. Write down 7 consecutive numbers so that the digit 2 is used exactly 16 times.

   There can be multiple answers. One possible solution is

   $$2215, 2216, 2217, 2218, 2219, 2220, 2221.$$

3. Three jumps of a two-headed dragon equals five jumps of a three-headed dragon. It takes a two-headed dragon the same amount of time to make four jumps as it takes a three-headed dragon to make seven jumps. Which of the dragons moves faster? Explain your answer.

   In terms of distance, 12 jumps of the two-headed dragon equals 20 jumps of three-headed dragon. But in terms of time, when the two-headed dragon makes 12 jumps, the three-headed dragon has already made 21 jumps, farther than the 20 jumps. So three-headed dragon is faster.

4. Two types of ladybugs live in the magical forest: some ladybugs have 6 dots, and the rest have 4 dots each. All the ladybugs with 6 dots always tell the truth. All the ladybugs with 4 dots always lie. You met several of these ladybugs.

   - The first ladybug told you: "All of us have the same number of dots."
   - The second ladybug said: "Altogether, we have 30 dots on our backs."
   - The third ladybug said: "No! Altogether, we have 26 dots on our backs."

   The rest of the ladybugs each said that only one of those three ladybugs told the truth. How many ladybugs did you meet?

   5 ladybugs in total; The first one is lying, the second one is lying, and the third one is telling the truth. All the rest of ladybugs are telling the truth.

5. Ben multiplied a number by 10 and got a prime number. Peter multiplied the same number by 15 and also got a prime number. Could it be that both of them did their computations correctly? Explain your answer.

   The only solution is $\frac{2}{5}$.

6. Solve the following riddle:

   Here is a riddle written on a cup:
   Eh is four times as much as **Oi**,
   Oh is four times as little as **Ai**,
   What do you get if you add all four of them up?

   Looking only at the first digit: h is the first digit of the number $4 \times i$, and i is the first digit of $4 \times h$. So h is the first digit of $16 \times h$, and the only two possibilities are

   $$h = 4, i = 6 \qquad \text{or} \qquad i = 4, h = 6.$$

   In the first case, $4 \times i = 24$, and thus $4 \times O + 2 = E$. Both E and O are single digits, so $O = 1, E = 6$, and

   $$Ai = Oh \times 4 = 14 \times 4 = 56.$$

   So $Eh + Oi + Oh + Ai = 64 + 16 + 14 + 56 = 150$.

   In the second case, $4 \times h = 24$, and thus $4 \times O + 2 = A$. Both A and O are single digits, so $O = 1, A = 6$, and

   $$Eh = Oi \times 4 = 14 \times 4 = 56.$$

   So $Eh + Oi + Oh + Ai = 56 + 14 + 16 + 64 = 150$.

   So in both cases, the answer is 150.

7. A dog and a cat are pulling a sausage in two different directions. If the dog takes a bite and run away, the cat will get 300 gr more than the dog. If the cat takes a bite and runs away, the dog will get 500 gr more than the cat. How much of the sausage will be left if each of them takes a bite and runs away?

   Suppose the dog's bite is d gr, and the cat's bite is c gr. Let the entire sausage be S gr. Then

$$\begin{cases} S - d = 300 + d \\ S - c = 500 + c \end{cases}$$

   By manipulating equations, we have $S - d - c = 400$.

8. Thirteen children were sitting around the table. All of the girls agreed that they will only tell the truth to each other and will lie to the boys. All of the boys agreed that they will only tell the truth to each other and lie to the girls. One of the children said to his/her neighbor on the right: "The majority of us are boys." The neighbor told his/her neighbor on the right: "The majority of us are girls," and so on, with the last child telling the first one: "The majority of us are boys." How many boys were there at the table?

   7 boys in total.