ORMC AMC 10/12 Group
Week 6: Practice Exam

November 5, 2023

Problem 1

Kate bakes a 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?

(A) 90 **(B)** 100 **(C)** 180 **(D)** 200 **(E)** 360

Problem 2

Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph (miles per hour), and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes?

(A) 64 **(B)** 65 **(C)** 66 **(D)** 67 **(E)** 68

Problem 3

In the expression $(\_\_\_\_ \times \_\_\_\_) + (\_\_\_\_ \times \_\_\_\_)$ each blank is to be filled in with one of the digits $1, 2, 3$, or 4, with each digit being used once. How many different values can be obtained?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 24

Problem 4

A three-dimensional rectangular box with dimensions X, Y, and Z has faces whose surface areas are $24, 24, 48, 48, 72$, and 72 square units. What is $X + Y + Z$?

(A) 18 **(B)** 22 **(C)** 24 **(D)** 30 **(E)** 36

Problem 5

How many subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ contain at least one prime number?

(A) 128 **(B)** 192 **(C)** 224 **(D)** 240 **(E)** 256

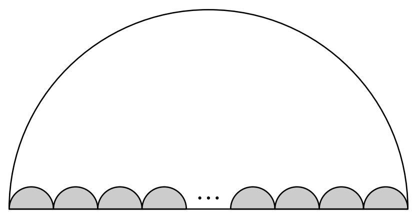
Problem 6

A box contains 5 chips, numbered $1, 2, 3, 4$, and 5. Chips are drawn randomly one at a time without replacement until the sum of the values drawn exceeds 4. What is the probability that 3 draws are required?

(A) $\frac{1}{15}$ **(B)** $\frac{1}{10}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{1}{4}$

Problem 7

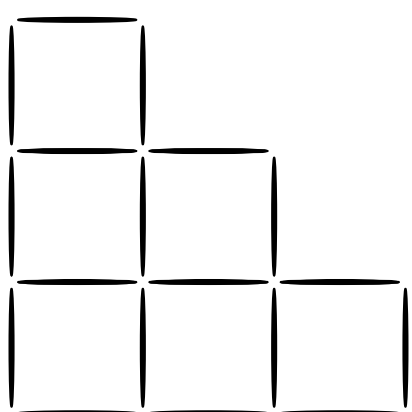
In the figure below, N congruent semicircles are drawn along a diameter of a large semicircle, with their diameters covering the diameter of the large semicircle with no overlap. Let A be the combined area of the small semicircles and B be the area of the region inside the large semicircle but outside the small semicircles. The ratio $A : B$ is $1 : 18$. What is N?



(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 36

Problem 8

Sara makes a staircase out of toothpicks as shown:



This is a 3-step staircase and uses 18 toothpicks. How many steps would be in a staircase that used 180 toothpicks?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 24 **(E)** 30

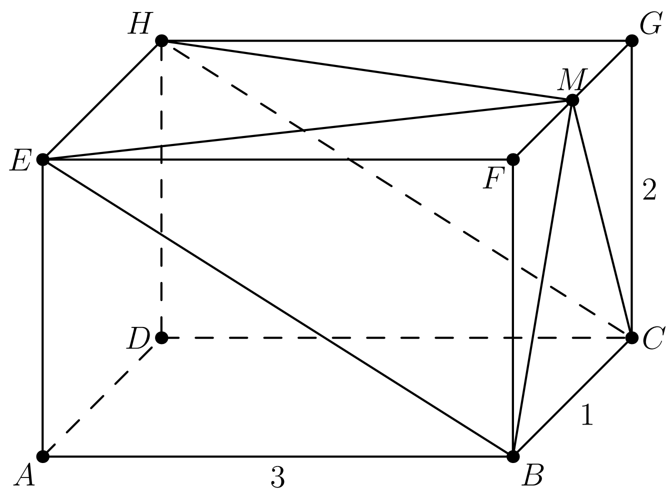
Problem 9

The faces of each of 7 standard dice are labeled with the integers from 1 to 6. Let p be the probability that when all 7 dice are rolled, the sum of the numbers on the top faces is 10. What other sum occurs with the same probability p?

(A) 13 **(B)** 26 **(C)** 32 **(D)** 39 **(E)** 42

Problem 10

In the rectangular parallelepiped shown, $AB = 3$, $BC = 1$, and $CG = 2$. Point M is the midpoint of $\overline{FG}$. What is the volume of the rectangular pyramid with base $BCHE$ and apex M?



(A) 1 **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{5}{3}$ **(E)** 2

Problem 11

Which of the following expressions is never a prime number when p is a prime number?

(A) $p^2 + 16$ **(B)** $p^2 + 24$ **(C)** $p^2 + 26$ **(D)** $p^2 + 46$ **(E)** $p^2 + 96$

Problem 12

Line segment $\overline{AB}$ is a diameter of a circle with $AB = 24$. Point C, not equal to A or B, lies on the circle. As point C moves around the circle, the centroid (center of mass) of $\triangle ABC$ traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?

(A) 25 **(B)** 38 **(C)** 50 **(D)** 63 **(E)** 75

Problem 13

How many of the first 2018 numbers in the sequence $101, 1001, 10001, 100001, \ldots$ are divisible by 101?

(A) 253 **(B)** 504 **(C)** 505 **(D)** 506 **(E)** 1009

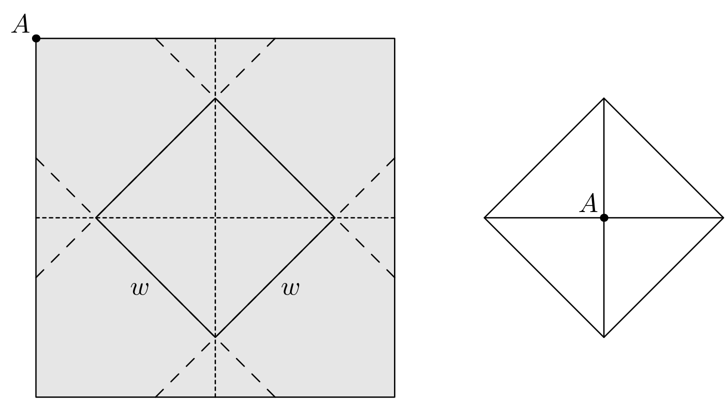
Problem 14

A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

(A) 202 **(B)** 223 **(C)** 224 **(D)** 225 **(E)** 234

Problem 15

A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point A in the figure on the right. The box has base length w and height h. What is the area of the sheet of wrapping paper?



(A) $2(w+h)^2$ **(B)** $\frac{(w+h)^2}{2}$ **(C)** $2w^2+4wh$ **(D)** $2w^2$ **(E)** w^2h

Problem 16

Let $a_1, a_2, \ldots, a_{2018}$ be a strictly increasing sequence of positive integers such that

$$a_1 + a_2 + \cdots + a_{2018} = 2018^{2018}.$$

What is the remainder when $a_1^3 + a_2^3 + \cdots + a_{2018}^3$ is divided by 6?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Problem 17

In rectangle $PQRS$, $PQ = 8$ and $QR = 6$. Points A and B lie on $\overline{PQ}$, points C and D lie on $\overline{QR}$, points E and F lie on $\overline{RS}$, and points G and H lie on $\overline{SP}$ so that $AP = BQ < 4$ and the convex octagon $ABCDEFGH$ is equilateral. The length of a side of this octagon can be expressed in the form $k + m\sqrt{n}$, where k, m, and n are integers and n is not divisible by the square of any prime. What is $k + m + n$?

(A) 1 **(B)** 7 **(C)** 21 **(D)** 92 **(E)** 106

Problem 18

Three young brother-sister pairs from different families need to take a trip in a van. These six children will occupy the second and third rows in the van, each of which has three seats. To avoid disruptions, siblings may not sit right next to each other in the same row, and no child may sit directly in front of his or her sibling. How many seating arrangements are possible for this trip?

(A) 60 **(B)** 72 **(C)** 92 **(D)** 96 **(E)** 120

Problem 19

Joey and Chloe and their daughter Zoe all have the same birthday. Joey is 1 year older than Chloe, and Zoe is exactly 1 year old today. Today is the first of the 9 birthdays on which Chloe's age will be an integral multiple of Zoe's age. What will be the sum of the two digits of Joey's age the next time his age is a multiple of Zoe's age?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Problem 20

A function f is defined recursively by $f(1) = f(2) = 1$ and

$$f(n) = f(n-1) - f(n-2) + n$$

for all integers $n \geq 3$. What is $f(2018)$?

(A) 2016 **(B)** 2017 **(C)** 2018 **(D)** 2019 **(E)** 2020

Problem 21

Mary chose an even 4-digit number n. She wrote down all the divisors of n in increasing order from left to right: $1, 2, \ldots, \frac{n}{2}, n$. At some moment Mary wrote 323 as a divisor of n. What is the smallest possible value of the next divisor written to the right of 323?

(A) 324 **(B)** 330 **(C)** 340 **(D)** 361 **(E)** 646

Problem 22

Real numbers x and y are chosen independently and uniformly at random from the interval $[0, 1]$. Which of the following numbers is closest to the probability that x, y, and 1 are the side lengths of an obtuse triangle?

(A) 0.21 **(B)** 0.25 **(C)** 0.29 **(D)** 0.50 **(E)** 0.79

Problem 23

How many ordered pairs (a, b) of positive integers satisfy the equation

$$a \cdot b + 63 = 20 \cdot \operatorname{lcm}(a, b) + 12 \cdot \gcd(a, b),$$

where $\gcd(a, b)$ denotes the greatest common divisor of a and b, and $\operatorname{lcm}(a, b)$ denotes their least common multiple?

(A)0 **(B)**2 **(C)**4 **(D)**6 **(E)**8

Problem 24

Let $ABCDEF$ be a regular hexagon with side length 1. Denote by X, Y, and Z the midpoints of sides $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of $\triangle ACE$ and $\triangle XYZ$?

(A) $\frac{3}{8}\sqrt{3}$ **(B)** $\frac{7}{16}\sqrt{3}$ **(C)** $\frac{15}{32}\sqrt{3}$ **(D)** $\frac{1}{2}\sqrt{3}$ **(E)** $\frac{9}{16}\sqrt{3}$

Problem 25

Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. How many real numbers x satisfy the equation $x^2 + 10,000\lfloor x \rfloor = 10,000x$?

(A) 197 **(B)** 198 **(C)** 199 **(D)** 200 **(E)** 201