

NUMBER BASES (TAKE 10)

MATH CIRCLE (ADVANCED) 10/28/2012

Today we still study some specific number bases:

Binary (base 2) with digits 0, 1.

Ternary (base 3) with digits 0, 1, 2.

Octal (base 8) with digits 0, 1, 2, 3, 4, 5, 6, 7.

Hexidecimal (base 16) with digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *A, B, C, D, E, F*.

0) Explain the algorithm presented to convert from base 10 to base n .

1) Fill in the following chart. Hint: There may be a way to save time!

Decimal	Binary	Ternary	Octal	Hexidecimal
59				
	1011001			
		12012		
				<i>AD</i>

2) a) Explain the trick for converting between Binary, Octal, and Hexidecimal.

b) Why can't you do a similar trick for Ternary?

3) Do the following conversions. Hint: You don't need to do much calculation!

a) 1025 from decimal to binary.

b) 4095 from decimal to hexadecimal.

c) 1330 from octal to ternary.

4) Do the following calculations:

a) $4096_{10} \times 512_{10}$ in hexadecimal.

b) $129_{10} \times 31_{10}$ in binary.

c) $242_{10} \times 80_{10}$ in ternary.

5) Prove that every natural number can be written as the difference of two numbers whose ternary representations contain only 0's and 1's.

6) Suppose I have four boxes with numbers (between 1 and 15) inside them.

A: 1, 3, 5, 7, 9, 11, 13, 15

B: 2, 3, 6, 7, 10, 11, 14, 15

C: 4, 5, 6, 7, 12, 13, 14, 15

D: 8, 9, 10, 11, 12, 13, 14, 15

If you pick a number between 1 and 15 and tell me all of the boxes it is in, I can tell you the number (try this if you want!). How do I do it?

7) What is the minimum number of weights which enable us to weigh any integer number of grams of gold from 1 to 100 on a standard balance with two pans? Weights may be placed only on the left pan.

8)* Repeat 7) if the weights can be placed on either side of the pan?

9)* Suppose $P(x)$ is an unknown polynomial, of unknown degree, with nonnegative integer coefficients. You have access to an oracle that, given an integer n , spits out $P(n)$, the value of the polynomial at n . However, the oracle charges a fee for each such computation, so you want to minimize the number of computations you ask the oracle to do. Show that it is possible to uniquely determine the polynomial after only two consultations of the oracle.

Some problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”