Intermediate 1 **Oct 29, 2023**

Think Outside the Box!

1. Peter exchanges stickers with his friends. For every sticker he gives someone, he gets 5 stickers back. Suppose he starts the exchange with just one sticker. How many stickers will he have after 30 exchanges?

2. Write down 7 consecutive numbers so that the digit 2 is used exactly 16 times.

3. Three jumps of a two-headed dragon equals five jumps of a three-headed dragon. It takes a two-headed dragon the same amount of time to make four jumps as it takes a three-headed dragon to make seven jumps. Which of the dragons moves faster? Explain your answer.

4. Two types of ladybugs live in the magical forest: some ladybugs have 6 dots, and the rest have 4 dots each. All the ladybugs with 6 dots always tell the truth. All the ladybugs with 4 dots always lie. You met several of these ladybugs.

   - The first ladybug told you: "All of us have the same number of dots."
   - The second ladybug said: "Altogether, we have 30 dots on our backs."
   - The third ladybug said: "No! Altogether, we have 26 dots on our backs."

The rest of the ladybugs each said that only one of those three ladybugs told the truth. How many ladybugs did you meet?

5. Ben multiplied a number by 10 and got a prime number. Peter multiplied the same number by 15 and also got a prime number. Could it be that both of them did their computations correctly? Explain your answer.

6. Solve the following riddle:

Here is a riddle written on a cup:
Eh is four times as much as **Oi**,
Oh is four times as little as **Ai**,
What do you get if you add all four of them up?

7. A dog and a cat are pulling a sausage in two different directions. If the dog takes a bite and run away, the cat will get 300 gr more than the dog. If the cat takes a bite and runs away, the dog will get 500 gr more than the cat. How much of the sausage will be left if each of them takes a bite and runs away?

8. Thirteen children were sitting around the table. All of the girls agreed that they will only tell the truth to each other and will lie to the boys. All of the boys agreed that they will only tell the truth to each other and lie to the girls. One of the children said to his/her neighbor on the right: "The majority of us are boys." The neighbor told his/her neighbor on the right: "The majority of us are girls," and so on, with the last child telling the first one: "The majority of us are boys." How many boys were there at the table?

9. The Big Island and The Small Island are both rectangular in shape and are divided into several rectangular counties. Each county has a road along one of the rectangle's diagonals. On each of the islands, these roads form a closed path which does not go twice through any of the points. Here is a map of the Small Island:

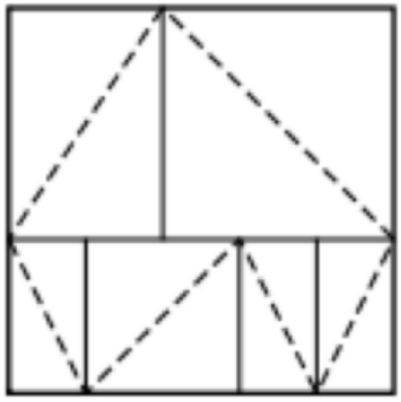


Figure 1: Small Island

Draw a possible map of the Big Island if you know that it has an odd number of counties. How many counties does your island have?

10. Thirty three giants are guarding a cave. The Wicked Witch agreed to pay them 240 gold coins under the following conditions:

    - The Wicked Witch divides the giants into several troops and pays each of the troops separately;
    - Within each of the troops, the coins are divided equally between the giants, and the remainder is given back to the Wicked Witch.

    (a) What is the biggest number of coins that the Wicked Witch can guarantee to herself if she can give the troops different numbers of coins (Note: the total number of coins given still must be 240)?

(b) What if she has to give each troop the same number of coins (independently of how many people are in each of the troops)?

11. Put signs of mathematical operations and parentheses in such a way that you get a true statement:

$$\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad = \quad 2 \tag{1}$$

$$4 \quad 4 \quad 4 \quad = \quad 6 \tag{2}$$

$$8\,8\,8\,8\,8\,8\,8\,8 \quad = \quad 1000 \tag{3}$$

12. To complete a job, it takes:

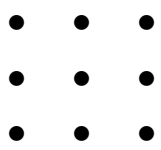
    - 2 hours for Alice and Bob working together;
    - 3 hours for Alice and Charlie working together;
    - 4 hours for Bob and Charlie working together.

    Suppose that all of them work at constant pace, how long does it take for Alice, Bob, and Charlie to finish this job together?

13. The following problem is attributed to Sir Isaac Newton: 70 cows eat the grass on a field in 24 days. 60 cows eat the grass on the same field in 30 days. How many cows would it take to eat all the grass in 96 days? (Hint: the grass continues to grow at a constant rate while the cows are eating it).

14. Alice borrowed 50 dollars from her mom and 50 dollars form her dad. So she had 100 dollars in total. She bought a skirt which costs 97 dollars, and she had 3 dollars left. She then gave 1 dollar to her mom and 1 dollar to her dad, and kept 1 dollar for herself. Now she owes her mom 49 dollars, owes her dad 49 dollars. That's 98 dollars in total, plus the 1 dollar she kept is 99 dollars in total. Where is the missing 1 dollar?

15. What is the minimum number of points which have to be removed from the following diagram so that in the remaining picture no three points lie in one line?



16. A student who is interested in geometry has a collection of $1\times1\times1$ dice. Each die has a certain colour. It wants to make a $3\times3\times3$ cube out of the dice so that small dice that meet at the very least on one corner are always of a different colour. What is the smallest amount of colours it needs to use?

17. In a bag there are only red and green marbles. If one randomly takes out five marbles, there is at least one red one. If one randomly takes out six marbles, there is at least one green one. What is the maximum number of marbles in the bag?

18. The numbers from 1 to 10 are written 10 times each on a board. Now the children play the following game: One child deletes two numbers off the board and writes instead the sum of the two numbers minus 1. Then a second child does the same, and so forth until there is only one number left on the board. What is the last number left?

19. What is the largest possible number you can get from the expression

$$1 * 2 * 3 * 4 * 5 * 6$$

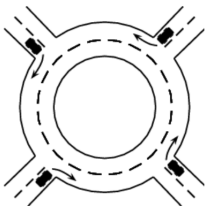
where each $*$ is replaced with either $+$ or $\times$. (You are not allowed to add parathesis).

20. Aron, Ben and Carl always lie. Each of them picks a red or a green stone.

    - Aron says: "My stone has the same colour as Bens stone."
    - Ben says: "My stone has the same colour as Carls stone."
    - Carl says: "Exactly two of us have red stones."

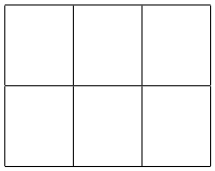
    What stone does each of them have?

21. Four cars drive into a roundabout at the same point in time, each one coming from a different direction:



    No car drives all the way around the roundabout, and no two cars leave at the same exit. In how many different ways can the cars exit the roundabout?

22. (**Challenge**) Every number among 1, 2, 3, 4, 5, 6 is written into exactly one cell of a 2 x 3 table:



In how many ways can this be done so that the sum of the numbers in every column and every row is divisible by 3?

23. (**Challenge**) You have 1000 bottles of wine for a birthday party. 20 hours before the party, the winery indicate 1 bottle of wine is filled with poison, without telling you which bottle. You have some lab mice, and you can test the wine by feeding wine samples to the mice. If you feed a mouse with poisonous wine, it will dies after 18 hours. (This means that you don't have time to do a second-round of testing if a mouse survive after drinking a bottle of wine). What is the minimum number of mice you would need to find out which bottle is poisonous?

24. (**Challenge**) Alice and Bob play a game on a 4 by 4 grid. On his or her turn, a player chooses a number not yet appearing on the grid and writes it in an empty square of the grid. Alice goes first and then the players alternate. When all squares have numbers written in them, in each row, the square with the greatest number is colored black. Alice wins if she can then draw a line from the top of the grid to the bottom of the grid that stays in black squares, and Bob wins if she can't. (Lines can be vertical or diagonal.) Find a wining strategy for Bob.